Corporate Real Decisions and Seasonalities in Stock and Accounting Data*

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This Version: April 19, 2022

Abstract

We develop a real options asset pricing model in which corporate real decisions can translate exogenous seasonality in state variables into endogenous expected return seasonality. Assuming exogenous output price seasonality, the model considers a firm able to produce, store in inventory, and sell out of inventory some output good, separating the production and selling decisions. The model suggests that a firm with low inventory holding costs optimally produces to build up inventories over some period before its high price season and sells its entire inventories in that season. Heeding that strategy, the firm gradually prepays quasi-fixed costs over that period, inducing its operating leverage and thus expected return to drop. Conversely, a firm with high inventory holding costs optimally produces much closer to its high price season, leading its expected return to be more stable over time. Our empirical work shows that our model significantly aids our understanding of recent stock seasonality anomalies.

Keywords: Asset pricing, real options, seasonalities, dynamic operating leverage, output inventories.

JEL classification: G11, G12, G13, G14.

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*We are indebted to Michael Brennan, Sungjun Choi, Hassan Ilyas, Gaurav Kankanhalli, Matti Keloharju, Bart Lambrecht, Peter Nyberg, Mark Shackleton, Greg Tindall, and Paul Zimmermann as well as conference participants at the 2021 British Academy of Management (BAM) Conference (online), the 2021 International Young Finance Scholars Conference (online), the 2021 New Zealand Finance Meeting (online), the 2022 Eastern Finance Association Annual Meeting (Washington), the 2022 French Finance Association International Conference (Saint Malo, expected), the 2022 European Financial Management Association (EFMA) Annual Meeting (Rome, expected), and the 2022 Financial Management Association (FMA) European Conference (Lyon, expected) for insightful and constructive suggestions.

1 Introduction

A large empirical literature in economics and finance documents seasonalities in stock, accounting, and macroeconomic variables. Chang et al. (2017) and Hartzmark and Solomon (2018), for example, find seasonalities in corporate earnings. Heston and Sadka (2008, 2010) and Keloharju et al. (2016, 2021) report seasonalities in stock returns, showing that a stock's past returns over the same calendar month positively signal future returns ("same-calendar-month premium"). Ogden (2003) discovers seasonalities in aggregate production, consumption, and market capitalization. Interestingly, Grullon et al. (2020) connect the seasonalities in stock and accounting data, establishing that stocks tend to underperform (outperform) in their high (low) sales quarters ("seasonal sales premium").

In this paper, we develop a real options model in which a firm exposed to seasonal variations in its output price is able to produce, store in inventory, and sell out of inventory some homogenous output good, separating the production and selling decisions. The model predicts that the optimal policy for a firm with low inventory holding costs is to spread out production over some period before its high price season, store the produced output in inventory, and sell it in the high price season, in that way minimizing its convexly increasing production costs. An interesting asset pricing implication of that policy is that such a firm essentially prepays some of its quasi-fixed costs and thus lowers its operating leverage up until its high price season. In turn, the gradual decline in operating leverage leads to a gradual decline in the firm's expected return up until that season. Conversely, the model predicts that the optimal policy for a firm with high inventory holding costs is to produce closer to its high price season, leading the expected return of such a firm to be more stable over time. In agreement with those predictions, we empirically establish that a firm's ability to build up inventories strongly and positively conditions Grullon et al.'s (2020) seasonal sales premium.

To offer intuition for why we suspect inventory holding costs to vary across firms, consider the Rhodes Island toy manufacturer Hasbro and the Californian lemon producer Limoneira. While both firms generate highly seasonal sales, with Hasbro making the largest fraction of its sales over Xmas and Limoneira over the lemon harvest season, only Hasbro but not Limoneira builds up significant output inventories before its high sales season. To wit, while Hasbro's output inventory holdings tend to be almost 30% higher than their annual average before its high sales quarter, Limoneira's holdings only tend to be about 10% higher. The reason is obviously that it is cheaper and more feasible to store durable goods (like toys) than non-durable goods (like lemons), inducing Hasbro to spread out its production over a much longer period before its high sales season than Limoneira.

In our theoretical work, we develop a real options model of an all-equity-financed firm operating over an infinite horizon and facing a stochastic output price obeying a *generalized* geometric Brownian motion. Crucially, the drift term of that geometric Brownian motion contains a sine function producing seasonal variations in the output price. In each instant, the firm optimally decides how much output to produce and add to its inventory ("production decision") and how much to sell out of its inventory ("selling decision"). The model solution suggests that the firm's optimal policy is to produce to build up output inventories over some period before its high-price season, to sell its entire output inventory in that season, and to produce to instantaneously sell over some period after that season. Importantly, inventory holding costs critically condition the lengths of the inventory building and the instantaneous selling periods, with higher costs leading to a longer instantaneous selling but a shorter inventory building period. Yet, since the firm only prepays quasi-fixed costs over the inventory building period, it only observes gradual declines in its operating leverage and expected return over that period. The upshot is that a greater ability to build up inventories, as facilitated through lower inventory holding costs, leads exogenous seasonal variations in output prices to more strongly translate into positively (negatively) related endogenous seasonal variations in sales (expected firm returns).

We also look into an extension of our model in which we award the firm a growth option allowing it to expand its production capacity. While the extension demonstrates that, in a world with seasonal output prices, corporate investment can also be seasonal, it further suggests that the growth option does not greatly modify how seasonality in the output price translates into seasonality in the expected firm return, at least not when the firm does not become more or less likely to exercise the growth option over time. The main lesson to take away is that our main theoretical conclusions are driven by the firm's production and selling policies, and not by its investment policies.

We next empirically test our model's main novel prediction that the ability to build up output inventories acts as the mechanism negatively translating seasonal variations in the output price/sales into seasonal variations in stock returns. To facilitate our tests, we follow Grullon et al. (2020) and calculate a firm's historical ratio of sales over a quarter to annual sales, first using the variability in that ratio over the year to distinguish between seasonal (high variability) and non-seasonal (low variability) firms and next using the ratio itself to identify the firm's high sales season. Similarly, we calculate a firm's historical ratio of inventory holdings at the end of some quarter to average inventory holdings over the ends of all quarters in a year to determine whether the firm holds abnormally high inventories at the end of the quarter. We finally interpret those firms with high (low) inventory holdings at the end of the quarter before their high (low) sales quarter as "inventory builders," while we interpret those with less high (less low) inventory holdings at that time as "non-inventory-builders."

Our empirical evidence broadly supports our theory. In line with Grullon et al. (2020), we find that seasonal firms earn significantly lower stock returns in their high sales relative to their low sales quarters. Our portfolio sorts, for example, suggest that their mean monthly value-weighted return is only 0.45% in their high sales quarters but 0.96% in their low sales quarter. The difference of –0.51% is highly significant (t-statistic: –3.64). Similarly, Fama and MacBeth (1973, FM) regressions of those firms' stock returns on the historical quarterly sales-to-annual sales ratio and controls also yield a significantly negative seasonal sales premium. More crucially, we next demonstrate that inventory building strongly conditions how sales seasonality translates into stock return seasonality. While seasonal inventory builders, for example, generate a significant spread in their mean monthly value-weighted returns across their high and low sales quarters of –0.91% (t-statistic: –5.82), the same spread is an insignificant –0.27% (t-statistic: –0.83) for non-inventory-builders. Similarly, while subsample FM regressions conducted on seasonal inventory builders yield a seasonal sales premium of –0.62% per month (t-statistic: –4.25), those same regressions conducted on seasonal non-inventory builders yield a corresponding seasonal sales premium of only –0.20% (t-statistic: –1.79).

While our main evidence relies on total inventory holdings to proxy for output ("finished goods") holdings, we next establish that output and total inventory holdings are highly positively correlated over the sample period for which both inventory holding variables are available (April 2008 to December 2019). Also, repeating our empirical analysis using output inventory holdings over the twelve-year period above does not alter our results. Excluding January observations, we finally confirm that our conclusions are not attributable to the January effect (Rozeff and Kinney (1976)).

We add to a recent literature on seasonalities in firm-level stock and accounting data. Chang et al. (2017) report that the announcement of high (low) earnings in fiscal quarters with historically high (low) earnings leads to high (low) abnormal returns, attributing those findings to investors and analysts overweighting the information in the most recent two to three earnings announcements and neglecting seasonal patterns in earnings. Heston and Sadka (2008, 2010) and Keloharju et al. (2016, 2021) establish that a stock's past same-calendar-month returns positively predict its future returns. While Keloharju et al. (2021) rationalize those findings using temporary mispricing effects canceling out over the calendar year, Hirshleifer et al. (2020) explain them using a stock's exposure to seasonal aggregate investor mood. Grullon et al. (2020) establish that stocks tend to underperform (outperform) in their high (low) sales quarters, linking those findings to firms' investment behavior and financial leverage and investor inattention. We contribute to these studies by developing a real options asset pricing model detailing how and under what conditions exogenous seasonalities in output prices translate into endogenous seasonalities in sales and expected firm returns. We next also offer empirical evidence in complete agreement with the model's novel implications.

We further contribute to studies looking into the stock pricing of inventory variables. Thomas and Zhang (2002) show that annual inventory changes negatively price stocks. Using a partial equilibrium investment model, Belo and Lin (2012) and Jones and Tuzel (2013) argue that the negative pricing arises through firms with larger inventories facing lower capacity adjustment costs, making them more flexible and decreasing their expected returns. Chen et al. (2005) and Alan et al. (2014) report that inventory holdings also negatively price stocks, attributing their results to investors only slowly incorporating inventory information from financial reports into their stock valuations. Interpreting large inventory holdings as a signal that a firm has prepaid a significant fraction of its quasi-fixed costs, our real options asset pricing model suggests a novel dynamic operating leverage (rather than investment) explanation for the negative inventory holdings and changes premiums.

We further add to the real options asset pricing literature pioneered by Berk et al. (1999) and Gomes et al. (2003). While earlier models in that literature focus on discount rate risks, ours follows the more recent models of Carlson et al. (2004), Cooper (2006), Hackbarth and Johnson (2015), and Aretz and Pope (2018) in focusing on cash flow risks. In contrast to the other more recent models, ours, however, allows for seasonality in a firm's output price modeled through a sine function included in our stochastic process. A further difference is that our model separates the production and selling decisions, allowing the firm to produce output, store that output in inventory for some while, and sell it then. In comparison, the other models restrict the firm to instantaneously sell its output.

We organize our paper as follows. In Section 2, we develop a model in which a firm exposed to seasonal variations in its output price can build up output inventories to serve that seasonality. In Section 3, we run empirical tests of the model's main novel implication that the ability to build up inventories is necessary for seasonal variations in price to translate into seasonal variations in stock returns. Section 4 offers robustness check results. Section 5 sums up and concludes. The appendix contains mathematical proofs, an extension of the model to the case in which the firm owns a single growth option allowing it to expand its production capacity, and variable definitions.

2 Theoretical Analysis

In this section, we develop a real options model in which a firm exposed to seasonality in its output price is able to produce output, store that output in inventory, and sell it later. We start with stating the model's assumptions. We next derive the optimal production and selling policies before outlining how we numerically solve the model. We then discuss the model's implications. We finally offer an extension of the model in which we award the firm a single capacity expansion option.

2.1 Model Assumptions

Consider an all-equity-financed firm operating over the infinite time horizon $t \in [0, \infty)$. In each instant, the firm is able to produce a homogenous output good and sell it either then or later at a stochastic price, P_t. We assume that the output price follows a *generalized* geometric Brownian motion whose drift term exhibits seasonal variations modeled through an additive sine function component

$$\mathrm{d}P_t = (\alpha + \kappa \sin(\eta t))P_t \mathrm{d}t + \sigma P_t \mathrm{d}B_t, \tag{1}$$

where the constant α is the linear time trend, the constant $\kappa \geq 0$ controls the magnitude of seasonal fluctuations, the constant $\eta > 0$ governs the length of a seasonal cycle,[1] the constant $\sigma > 0$ is volatility, and B_t is a Brownian motion. Setting $\kappa = 0$, stochastic process (1) collapses to a standard geometric Brownian motion, as usually studied in real options asset pricing models in the literature. Assuming

[1]The length of a seasonal cycle is $2\pi/\eta$. Since we interpret a seasonal cycle as a year, we always set $\eta = 2\pi$.

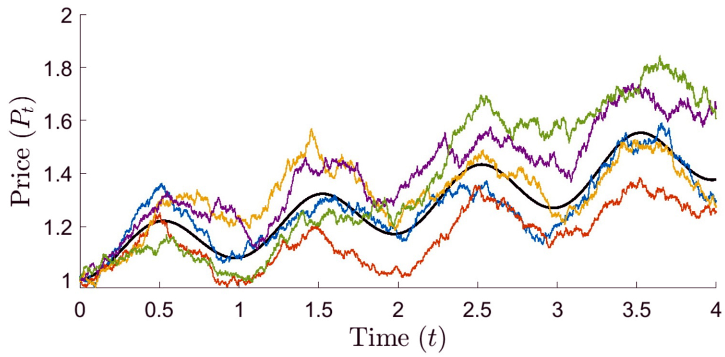

Figure 1: The figure plots five sample paths (non-black lines) and the time-0 conditional expectation (black line) of stochastic process (1) under the real world measure over the time period from $t = 0$ to 4. We describe the stochastic process parameter values in the first paragraph of Section 2.1.

an initial output price (P_0) of one, an annualized linear time trend (α) of 8%, a seasonal strength (κ) of 0.50, a periodicity (η) of 2π, and an annualized volatility (σ) of 10%, Figure 1 plots five sample paths (non-black lines) plus the time-0 expectation (black line) of stochastic process (1).

Consistent with Pindyck (1988) and Aretz and Pope (2018), the firm owns a fixed amount of installed capacity equal to $\bar{K} > 0$. In each instant, the firm is able to costlessly switch on each capacity increment to produce output, with one capacity unit producing one output unit per time unit. In accordance, the firm's output quantity at time t equals $Q_t \in [0, \bar{K}]$ per time unit. The firm incurs instantaneous costs from producing output determined by the convex function $C_P(Q_t) = c_1 Q_t + \frac{1}{2} c_2 Q_t^2$, where $c_1 \geq 0$ and $c_2 > 0$ are parameters. Having finished production, the firm instantaneously shifts each output increment into its output inventory, with the amount of output in inventory at time t equal to I_t. The firm pays a unit inventory cost of $c_I > 0$ per time unit, so that the present-value cost at time s from storing one output unit from that time until time t, $C_I(s, t)$, is

$$C_I(s,t) = \int_s^t e^{-ru} c_I \mathrm{d}u = \frac{c_I}{r}\left(e^{-rs} - e^{-rt}\right), \tag{2}$$

where r is the constant net risk-free rate of return. Finally, the firm is able to costlessly shift out of inventory and instantaneously sell an amount of output equal to $S_t \in [0, I_t]$ at each time t. As a consequence, the law of motion for the amount of output in inventory, I_t, is

$$\mathrm{d}I_t = (Q_t - S_t)\mathrm{d}t. \tag{3}$$

Setting $S_t = Q_t$ at each time t, the firm becomes an "instantaneous seller" of its concurrently produced output, as in other real options asset pricing models in the literature.

In a later extension, we also endow the firm with a single growth option enabling it to irreversibly double its production capacity $\bar{K}$ at an investment cost of k. To illustrate that our main theoretical conclusions are driven by the firm's production and selling (and not its investment) options, we, however, refrain from including the growth option in our initial derivations.

Overall, the main model thus contains two types of choice variables, the output quantities to be produced and shifted into inventory, Q_t, and the output quantities to be sold out of inventory, S_t, at each time t. The extended model adds the output price threshold at which the firm optimally exercises its growth option, P_t^*. In turn, the state space of both models is described by time t (due to the output price seasonality), the output price P_t, and the amount of output in inventory I_t.

2.2 Optimal Production and Selling Policies

We next find the firm's optimal production and selling policy by determining the optimal values for Q_t and S_t. To do so, we start with writing the firm's value at time t, $W(t, P_t, I_t)$, as

$$\begin{aligned} W(t,P_t,I_t) \quad &= \quad \max_{Q_{s;t},S_{u;s,t}} \mathbb{E}_t^{\mathbb{Q}}\left[\left(\int_t^{\infty}\left(-C_P(Q_{s;t})+\int_s^{\infty}\left(P_u e^{-r(u-s)}-C_I(s,u)\right)S_{u;s,t}\mathrm{d}u\right)e^{-r(s-t)}\mathrm{d}s\right)\right] \\ &\qquad \text{subject to } \int_t^{\infty} S_{u;t,t}\mathrm{d}u = Q_{t;t}+I_t \text{ and } \int_s^{\infty} S_{u;s,t}\mathrm{d}u = Q_{s;t}\ \forall s\in(t,\infty], \end{aligned} \tag{4}$$

where $Q_{s;t}$ is the output quantity produced at time s, $S_{u;s,t}$ are the sales out of that output quantity at time $u \geq s$, and $\mathbb{E}_t^{\mathbb{Q}}$ is the time-t conditional expectation under the equivalent martingale measure under which each asset's discounted value follows a martingale. To better understand Equation (4), fix s at $s^* \geq t$. We can then view $Q_{s^*;t}$ as the output quantity produced at time s^*, $-C_P(Q_{s^*;t})$ as the cost from producing that quantity at time s^*, and $\int_{s^*}^{\infty}\left(P_u e^{-r(u-s^*)} - C_I(s^*,u)\right)S_{u;s^*,t}\mathrm{d}u$ as the time-s^* present value from selling that quantity over the time period starting from s^*.

To determine the firm's optimal choices for $Q_{t;t}$ and $S_{u;t,t}$ (i.e., its optimal production and selling decisions at the current time t), we first notice that those choices do not affect the gains and costs associated with output produced at later times. The reason is that the current output choice $Q_{t;t}$ does neither affect the production nor the inventory holding costs associated with output produced in the future. In the same vein, the current selling decisions $S_{u;t,t}$ do not affect the revenue generated through selling output produced in the future.[2] The upshot is that we can find $Q_{t;t}$ and $S_{u;t,t}$ separately from the firm's choices about output produced in the future, solving the problem

$$\max_{Q_{t;t}, S_{u;t,t}} \mathbb{E}_t^{\mathbb{Q}} \left[-C_P(Q_{t;t}) + \int_t^{\infty} \left(P_u e^{-r(u-t)} - C_I(t,u) \right) S_{u;t,t} \mathrm{d}u \right], \tag{5}$$

subject only to the first (but not the second) constraint shown in Equation (4).

As a next step, we realize that if it were optimal to sell a non-zero amount of output at the current time t (i.e., if $S_{t;t,t} > 0$), it would also be optimal to sell all other output produced or already held in inventory at the same time since the present value from selling output net of inventory holding costs does not differ over the output increments. As a result, it is optimal to either sell no ($S_{t;t,t} = 0$) or all output ($S_{t;t,t} = Q_{t;t} + I_t$) at the current time t ("bang-bang solution"). To establish which of the two choices is optimal, we find the time $t^* \in [t, +\infty)$ maximizing the present value from selling a single output unit net of inventory holding costs, solving the problem

$$\max_{u \in [t,\infty)} \mathbb{E}_t^{\mathbb{Q}}[P_u] e^{-r(u-t)} - C_I(t,u) \tag{6}$$

and setting $S_{t;t,t}$ to $Q_{t;t} + I_t$ if $t = t^*$ and else to zero. We show the first-order condition of problem (6) in the appendix, noting that it has to be numerically solved for t^*.

Having determined its optimal selling policy, the firm's production problem becomes

$$\max_{Q_t \in [0,\bar{K}]} -C_P(Q_{t;t}) + \left(\mathbb{E}_t^{\mathbb{Q}}[P_{t^*}] e^{-r(t^*-t)} - C_I(t,t^*) \right) Q_t, \tag{7}$$

implying that the optimal output quantity produced at the current time t and sold, in expectation, at time t^*, Q_t^*, maximizes the discounted benefit from production, the present value of the sales revenue net of inventory costs, minus the production costs. We show the optimal Q_t^* in the appendix.

Importantly, we stress that the firm does not pre-commit to selling output at the time $t^* > t$. In each instant, the firm decides whether it is optimal to sell all output now (i.e., $t^* = t$) or later (i.e.,

[2] More technically, P_u, $C_I(s,u)$, and $C_P(Q_s)$, with $s > t$ and $u > s$, are independent of the firm's choices up until time t. The independence of P_t comes from the fact that the firm is a price-taker (i.e., the output price does not depend on the amount of quantity sold by the firm). The independence of $C_I(s,u)$ comes from the fact that total inventory holding costs are linear in the amount of output held in inventory (see Equation (2)).

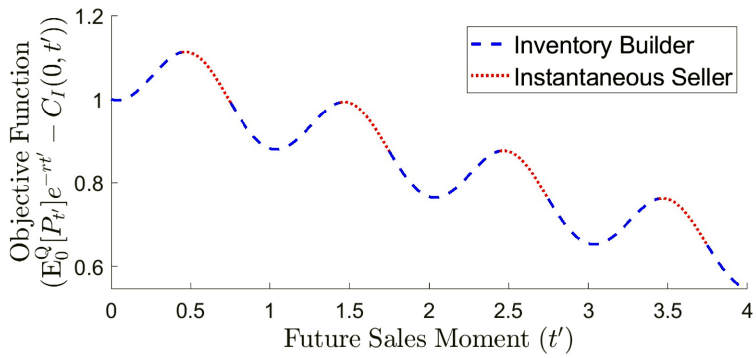

Figure 2: The figure plots the time-0 present value of the revenue from selling one output unit at future time t' minus the present value of storing it until then (Equation (6)). We describe the parameter values in the first paragraph of Section 2.1 and in the penultimate paragraph of Section 2.2.

$t^* > t$). Even when the firm's initial intention was to sell at some specific time, it is still able to alter that decision and sell earlier or later in response to variations in the output price.

Figure 2 helps us to better understand the firm's optimal policies. Relying on the same stochastic process parameter values as in Section 2.1 and assuming an expected output-price mimicking portfolio return (μ) of 10%, a risk-free rate of return (r) of 3%, and an inventory cost (c_I) of 10%, all per annum, the figure plots the time-0 present value from selling one output unit net of inventory holding costs at future time $t' \in [t, \infty)$ over the period from $t = 0$ to 4. If the present value attains its global maximum at the current time, the firm optimally sells its entire instantaneously-produced and in-inventory output at that time. If it attains its global maximum later, the firm instead builds up its output inventory without selling output. In the figure, the firm expects to build up its output inventory without selling until about time $t = 0.48$ to sell its entire output inventory then. Conversely, it expects to produce to instantaneously sell from about time $t = 0.48$ to 0.72, before again building up its output inventory without selling from about time $t = 0.72$ to 1.42, and so on.

For simplicity, we refer to a firm building up its output inventory without selling as an "inventory builder" and to a firm producing to instantaneously sell as an "instantaneous seller."

2.3 Market Value and Expected Excess Return

We next value the firm using contingent claims analysis. To do so, let $W^{\mathrm{IB}}(t, P_t, I_t)$ denote the time-t value of the firm conditional on an output price of P_t and an amount of output in inventory equal to I_t in states in which it is optimal for the firm to act as an inventory builder. Conversely, let $W^{\mathrm{IS}}(t, P_t)$

denote that same value conditional on an output price of P_t in states in which it is optimal for the firm to act as an instantaneous seller. Assuming complete markets, we are able to show that $W^{\text{IB}}(t, P_t, I_t)$ satisfies the three-dimensional partial differential equation (PDE)

$$\frac{\partial W^{\text{IB}}}{\partial t} + Q_t^* \frac{\partial W^{\text{IB}}}{\partial I_t} + (r - \delta_t) P_t \frac{\partial W^{\text{IB}}}{\partial P_t} + \frac{1}{2}\sigma^2 P_t^2 \frac{\partial^2 W^{\text{IB}}}{\partial P_t^2} - rW^{\text{IB}} - c_1 Q_t^* - \frac{1}{2} c_2 Q_t^{*2} - c_I I_t = 0, \quad (8)$$

subject to boundary conditions stated in Appendix A. The inhomogeneity in PDE (8), $-c_1 Q_t^* - \frac{1}{2} c_2 Q_t^{*2} - c_I I_t$, are the instantaneous production and inventory cost cash outflows at time t. Conversely, we are also able to show that $W^{\text{IS}}(t, P_t)$ satisfies the two-dimensional PDE

$$\frac{\partial W^{\text{IS}}}{\partial t} + (r - \delta_t) P_t \frac{\partial W^{\text{IS}}}{\partial P_t} + \frac{1}{2}\sigma^2 P_t^2 \frac{\partial^2 W^{\text{IS}}}{\partial P_t^2} - rW^{\text{IS}} + P_t Q_t^* - c_1 Q_t^* - \frac{1}{2} c_2 Q_t^{*2} = 0, \quad (9)$$

subject to other boundary conditions also in Appendix A. The inhomogeneity in PDE (9), $P_t Q_t^* - c_1 Q_t^* - \frac{1}{2} c_2 Q_t^{*2}$, is now the instantaneous net cash flow from producing an amount of output equal to Q_t^* at time t and instantaneously selling that amount of output at the output price P_t.

To obtain the value of the firm, $W(t, P_t, I_t)$, we have to "knit together" the firm value components $W^{\text{IB}}(t, P_t, I_t)$ and $W^{\text{IS}}(t, P_t)$ at the times at which the firm optimally switches from acting as an inventory builder to acting as an instantaneous seller and vice versa, noticing that the set of optimal switching times depends on the output price. At the times when the firm optimally switches from inventory builder to instantaneous seller, we impose the value-matching condition

$$W^{\text{IB}}(t, P_t, I_t) = W^{\text{IS}}(t, P_t) + P_t I_t, \quad (10)$$

where $P_t I_t$ is the sales revenue generated from selling the entire output in inventory I_t at the output price P_t. In contrast, at the times when the firm optimally switches from instantaneous seller to inventory builder, we impose the corresponding value-matching condition

$$W^{\text{IS}}(t, P_t) = W^{\text{IB}}(t, P_t, I_t = 0), \quad (11)$$

where $W^{\text{IB}}(t, P_t, I_t = 0)$ is the value of the inventory builder at time t conditional on an output price

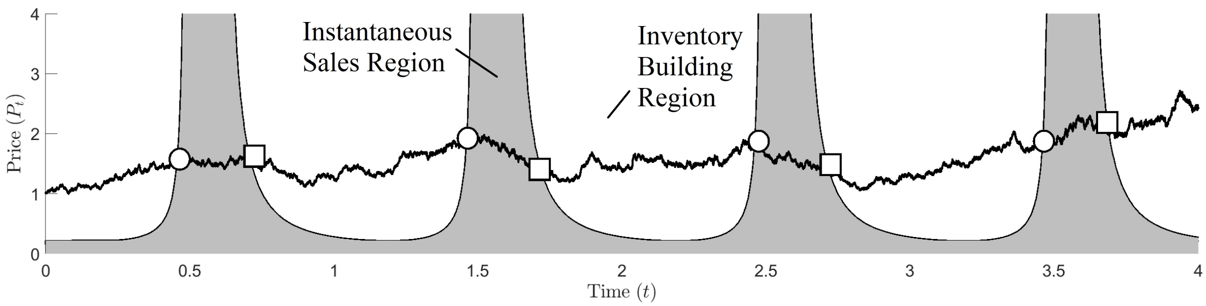

Figure 3: The figure plots the regions in the output price-time state space in which the firm optimally acts as inventory builder (white regions) and instantaneous seller (gray regions) over the time period from $t = 0$ to $t = 4$. It further plots one single output price sample path over the same period (solid black line), with the circles indicating when the firm switches from inventory builder to instantaneous seller and the squares indicating when it switches the other way round. We describe the parameter values in the first paragraph of Section 2.1, the penultimate paragraph of Section 2.2, and the penultimate paragraph of Section 2.3.

of P_t and an empty output inventory (i.e., $I_t = 0$). To avoid negative firm values, we further impose the general lower bound $P_t I_t + W^{\mathrm{IB}}(t, P_t, 0)$ in the inventory building region, which implies that the firm always immediately sells off its entire inventory when it is optimal to do so.

Since we are unable to solve the model in closed-form, we use an explicit finite difference method as solution technique. To do so, we first derive the set of optimal switching times conditional on the output price P_t. We then set up finite difference grids with sufficiently high maximum values for time t, price P_t, and output in inventory I_t. We now solve the two-dimensional grid for $W^{\mathrm{IS}}(t, P_t)$ assuming that the firm always acts like an instantaneous seller. We next solve the three-dimensional grid for $W^{\mathrm{IB}}(t, P_t, I_t)$ assuming that the firm acts as an inventory builder up until the final switching time, taking the boundary values for the final switching time from the solution to the prior two-dimensional grid. We then solve the two-dimensional grid for $W^{\mathrm{IS}}(t, P_t)$ assuming that the firm acts as instantaneous seller up until the penultimate switching time, taking the boundary values for the penultimate switching time from the solution to the following three-dimensional grid. We continue until we have dealt with all inventory building and instantaneous selling regions. See Appendix A for details.

Using the parameter values stated in Sections 2.1 and 2.2 and assuming production cost parameter values (c_1 and c_2) and an installed capacity ($\bar{K}$) of one, Figure 3 plot the inventory building (white) and instantaneous selling (gray) regions in the output price-time state space. It also plots one sample output price path. Interestingly, the figure shows that the firm always acts as instantaneous seller when the output price P_t is below a threshold of about 0.20. In accordance, it further suggests

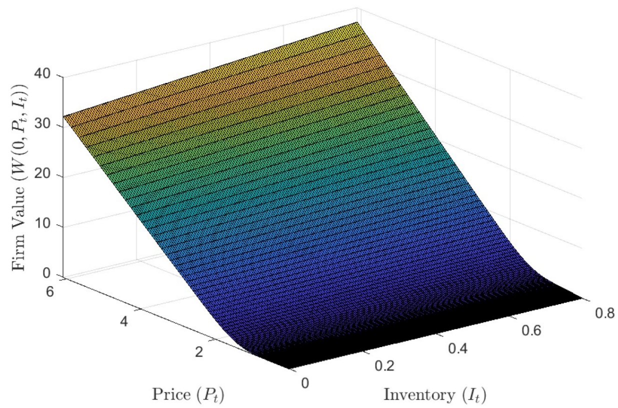

Figure 4: The figure plots the initial firm value, $W(0, P_t, I_t)$, against the output price, P_t, and the output in inventory, I_t. We describe the parameter values in the first paragraph of Section 2.1, the penultimate paragraph of Section 2.2, and the penultimate paragraph of Section 2.3.

that a higher price entices the firm to act more often as inventory builder, as can be seen from the inventory builder-to-instantaneous seller (instantaneous seller-to-inventory builder) switching time monotonically rising (dropping) with the output price. Intuitively, the greater propensity to act as inventory builder at higher output prices arises since a higher price implies greater price seasonality without changing inventory holding costs, with the low seasonality in the price below the threshold of about 0.20 never justifying incurring inventory holding costs. The figure finally plots all the times at which the firm switches from inventory builder to instantaneous seller (the circles) and all those at which it switches in the opposite direction (the squares) under the sample path.

Armed with the solution for the firm's value, we next compute the firm's expected return. Since the only state variable in our model requiring a risk premium is the output price, P_t, we can calculate the conditional expected excess firm return, $\mathbb{E}[r_W] - r$, from

$$\mathbb{E}[r_W] - r = \Omega_W(\mu - r), \tag{12}$$

where Ω_W is the elasticity of firm value $W(t, P_t, I_t)$ with respect to the output price P_t, and μ is the expected return of a mimicking portfolio whose value is perfectly positively correlated with the output price. Conversely, the elasticity is the ratio of the relative change in firm value to the relative change in the output price, $\Omega_W = \frac{\partial W(t,P_t,I_t)}{\partial P_t} \frac{P_t}{W(t,P_t,I_t)}$ (see Carlson et al. (2004, 2006, 2010), Cooper et al. (2005), Cooper (2006), Hackbarth and Johnson (2015), and Aretz and Pope (2018)).

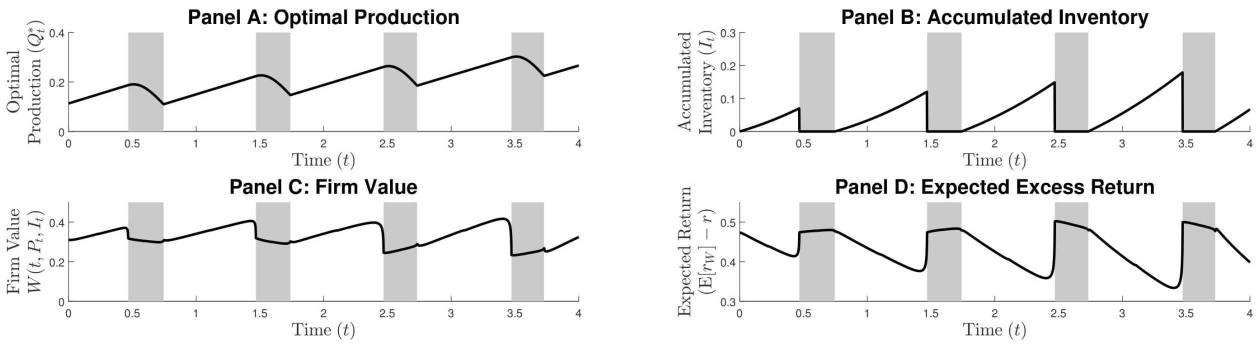

Figure 5: The figure plots the firm's optimal production quantity Q_t^* (Panel A), its accumulated output in inventory I_t (Panel B), its value $W(t, P_t, I_t)$ (Panel C), and its expected excess return $\mathbb{E}[r_W] - r$ (Panel D) over the period from $t = 0$ to 4 under an output price trajectory at which firm value shows no general tendency to rise or fall. The gray bars in each subplot indicate the periods during which the firm acts as an instantaneous seller. We describe the parameter values in the first paragraph of Section 2.1, the penultimate paragraph of Section 2.2, and the penultimate paragraph of Section 2.3.

2.4 Output Price Seasonality, Inventory Building, and Expected Return

We next investigate how inventory building shapes the relation between seasonality in the output price and seasonality in the expected firm return in our model. To do so, Figure 4 starts with plotting the initial firm value $W(0, P_t, I_t)$ against the output price P_t and inventory holding I_t conditional on the parameter values used in the prior sections. We stress that the firm is in an inventory building period at that time. In line with intuition, the figure suggests that the initial firm value rises monotonically with the output price because a higher output price shifts upward the distribution of future output price values. It further indicates that firm value also rises monotonically with the output in inventory since the present value of selling one output unit at the next optimal selling time exceeds the present value costs of holding the output unit in inventory until that optimal selling time.

Figure 5 displays the firm's optimal production decisions (Q_t^*; Panel A), corresponding output in inventory (I_t; Panel B), value ($W(t, P_t, I_t)$; Panel C), and expected excess return ($\mathbb{E}[r_W] - r$; Panel D) over the time period from $t = 0$ to 4 under an output price trajectory at which firm value stays roughly constant over time.[3] The gray bars in the panels indicate instantaneous selling periods. Panel A suggests that the firm raises its production quantity both over the entire period but also over each

[3] In theory, this output price trajectory is equal to the expected output price under the real-world measure with the linear drift rate, α, set to zero. Since we, however, use finite-time grids to solve our model, setting α to zero induces firm value to slightly decrease over time. To mitigate that issue, we set α equal to 0.03. Importantly, our conclusions do not depend on the specific sample path for the output price employed in the figures.

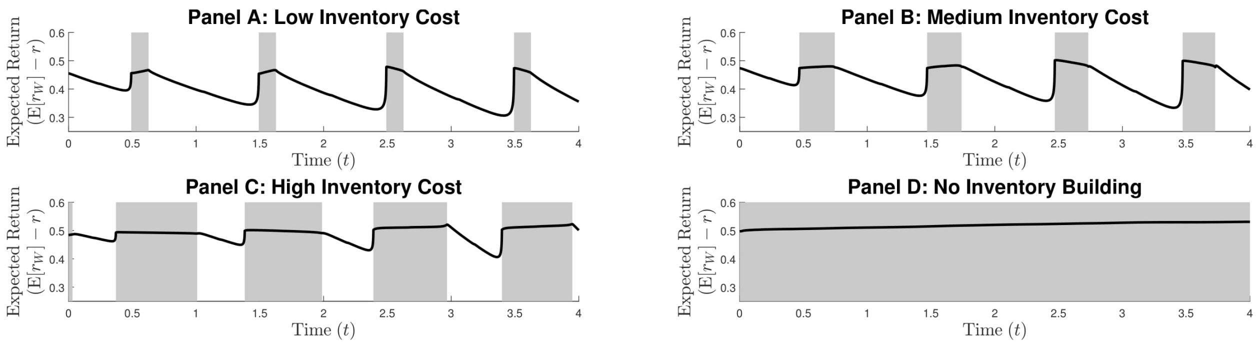

Figure 6: The figure plots the firm's expected excess return $\mathbb{E}[r_W] - r$ under an inventory holding cost of 0.01 ("low;" Panel A), 0.10 ("medium;" Panel B), 0.40 ("high;" Panel C), and infinity ("no inventory building;" Panel D) over the period from $t = 0$ to 4 under an output price trajectory at which firm value shows no general tendency to rise or fall. The gray bars in each subplot indicate the periods during which the firm acts as an instantaneous seller. We describe the parameter values in the first paragraph of Section 2.1, the penultimate paragraph of Section 2.2, and the penultimate paragraph of Section 2.3.

inventory building period. While the overall increase is due to the positive linear drift component of the output price (i.e., $\alpha = 0.03$), the increase over each inventory building period is attributable to the firm trading off the benefits from spreading production over time to minimize production costs and the costs from holding inventory.[4] In accordance, Panel B shows that the firm's output in inventory rises convexly over the inventory building periods, before jumping to zero at the start of an instantaneous selling period and staying there until the end of that period.

Panel C of Figure 5 demonstrates that the firm's optimal production and selling policies induce its value to sharply rise over inventory building periods but to stay more constant over instantaneous selling periods. The reason for the sharp rises over the inventory building periods is that the firm gradually pays off production plus inventory holding costs over those periods associated with output to be sold for revenue only at the end of the periods. The upshot is that the firm becomes gradually more operationally delevered over inventory building periods. In contrast, the firm simultaneously earns the revenues and pays the costs associated with output over the instantaneous selling periods, leading its operating leverage to be more stable over those periods. Since operating leverage positively conditions the expected excess return (see Carlson et al. (2004, 2006, 2010)), Panel D finally suggests that the expected excess return markedly decreases over the inventory building periods and toward the optimal selling date but stays more constant over the instantaneous selling periods.

[4]That the firm does not use its inventory building capabilities to smooth its production over time aligns with the empirical findings of Miron and Zeldes (1988) and Milne (1994).

The intuition behind the *dynamic* operating leverage effect in our model is that an inventory builder essentially prepays "quasi-fixed" costs incurred in producing and selling output in comparison to an instantaneous seller. While those costs do not affect how firm value responds to output price shocks, they still lower it, raising the expected firm return through the $P_t/W(t, P_t, I_t)$ component in the elasticity in Equation (12). Thus, our dynamic effect closely aligns with the static operating leverage effects included in other real options asset pricing models in the literature (see, e.g., Carlson et al. (2004, 2006, 2010), Cooper (2006), Hackbarth and Johnson (2015), and others).

Figure 6 evaluates how the magnitude of inventory holding costs (c_I) affects the relation between seasonality in the output price and seasonality in the expected firm return. To that end, we redo Panel D of Figure 5 under the assumption that c_I is 0.01 ("low;" Panel A), 0.10 ("medium;" Panel B), 0.40 ("high;" Panel C), and infinite ("no inventory building;" Panel D). Unsurprisingly, the gray bars suggest that a higher inventory holding costs induce the firm to optimally act less (more) often as inventory builder (instantaneous seller). Since the expected firm return is, however, almost stable over instantaneous selling periods, the higher costs also loosen the link between expected return and output price. When c_I is low, the expected return unambiguously drops until the optimal sales date, jumps up directly after that, and then almost directly drops again. Conversely, when c_I is higher, the expected return only drops over some period before the optimal sales date, jumps up directly after that, and then stays close to constant for some period. In the limiting case in which c_I is so high that the firm never builds up inventories, the expected return stays essentially constant over time.[5] The lesson to be learned is that lower inventory holding costs induce seasonality in the output price to more strongly translate into positively (negatively) related seasonality in sales (the expected firm return).

2.5 Endowing the Firm with a Capacity Expansion Option

We next study an extension of our model in which we award the firm a growth option allowing it to irreversibly double its production capacity at an investment cost of k in Appendix B. The appendix suggests that the firm is more likely to exercise the growth option earlier (rather than later) on in an

[5] We also ran comparative statics to gain further insights into the model's implications. In line with intuition, weaker seasonal output price fluctuations (lower κ), lower production costs (lower c_1 and/or c_2), and a lower production capacity (lower $\bar{K}$) all weaken the negative relation between seasonality in the output price and seasonality in the expected firm return since they either lower the firm's incentives or its ability to build up output inventories.

inventory building period, so that it can still use the new capacity to produce more output to be sold during the next high output price season. More technically, Figure B.1 in that appendix reveals that the optimal investment threshold P_t^* is itself seasonal, with it taking on higher (lower) values at the end (in the middle) of an inventory building period. Notwithstanding, Figure B.2 in that same appendix suggests that awarding the firm the growth option does not materially change how its expected excess return evolves over time, at least not when the firm stays at a close to constant distance from exercising the option. In particular, the figures shows that the expected excess return still markedly drops over inventory building periods, jumps up directly at their end, and stays close to constant over instantaneous sales periods, just like it did in Panel D of Figure 5.

2.6 Empirical Implications of our Theoretical Work

Taken together, we deduce the following implications from our theoretical work:

(1) A real options asset pricing model with seasonal sales and inventory building can generate a negative relation between the seasonal variations in a firm's output price and expected excess return if inventory holding costs are sufficiently low for firms to build up significant amounts of output inventories to be sold during the next high output price season.

(2) Since the model firm sells the lion share of its output on the date on which the output price reaches its seasonal high, the model also generates a negative relation between the seasonal variations in the firm's sales and expected excess return under the same conditions.

(3) Raising the inventory holding costs, the model implies that the firm builds up fewer inventories to be sold during the next high output price season, flattening the negative relation between the seasonal variations in a firm's output price (or: sales) and expected excess return. In fact, as inventory holding costs become sufficiently high, seasonal variations in the output price (or: sales) no longer translate into seasonal variations in the expected excess return.

While it is hard to empirically study implication (1) since we do not observe the prices at which firms sell output, Grullon et al. (2020) offer strong empirical support for implication (2). Conversely, we will offer strong empirical support for implication (3) in the next section.

3 Empirical Evidence

In this section, we empirically study our main novel theoretical implication that a greater ability to build up output inventories induces a more negative relation between seasonalities in sales and expected firm returns. We first introduce our analysis variables and data sources, with Table C1 in Appendix C offering more details about variable definitions. We next confirm Grullon et al.'s (2020) conclusion that single-stock returns tend to be negatively related to a firm's historical sales proportion in the current fiscal quarter ("seasonal sales premium"). Most crucially, we finally show that the negative relation between stock returns and historical sales proportions becomes more pronounced with the extent to which firms are able to build up output inventories, supporting our theory.

3.1 Variables, Data Sources, and Descriptive Statistics

We follow Grullon et al. (2020) in classifying firms as seasonal or non-seasonal firms and in finding the high and low sales quarters of seasonal firms. To be specific, we first calculate how much each fiscal quarter of a firm contributes to its annual sales for the fiscal year ending directly before the prior and for the fiscal year directly before that, ensuring that all data were available to real-time investors. We next take averages of the proportions by fiscal quarter to mitigate outliers. More technically, we calculate the average proportion for firm i in fiscal year y and quarter q, $QSales_{i,y,q}$, as:

$$QSales_{i,q,y} = \left(\frac{QuarterlySales_{i,q,y-2}}{AnnualSales_{i,y-2}} + \frac{QuarterlySales_{i,q,y-3}}{AnnualSales_{i,y-3}} \right) /2, \tag{13}$$

where $QuarterlySales_{i,q,y}$ are firm i's sales over quarter q of fiscal year y, and $AnnualSales_{i,y}$ are its total sales over that fiscal year. Intuitively, we can view a high (low) $QSales_{i,q,y}$ value as signalling that a firm made a high (low) proportion of its annual sales over the current fiscal quarter over the two fiscal years ending directly before the prior. To identify a firm as seasonal or non-seasonal, we finally calculate the standard deviation of $QSales_{i,q,y}$ over year y, labelling it $Seasonality_{i,y}$. Intuitively, we can view a high (low) $Seasonality_{i,y}$ value as signalling a seasonal (non-seasonal) firm.[6]

We measure inventory building using an approach analogous to that used to calculate $QSales_{i,q,y}$. In

[6] Our empirical conclusions are robust to excluding firm-fiscal year observations for which the sum of quarterly sales does not equal annual sales. Moreover, they are also robust to calculating $QSales$ and $Seasonality$ using a larger number of prior fiscal years (always excluding the most recent prior fiscal year).

particular, we first calculate the ratio of a firm's quarterly inventories at the end of some quarter to its average quarterly inventories over the fiscal year for the fiscal year directly ending before the prior and the fiscal year before that. We then again take averages of those ratios by fiscal quarter. More technically, we calculate the average for firm i in quarter q of fiscal year y, $QInventory_{i,q,y}$, as:

$$QInventory_{i,q,y} = \left(\frac{QuarterlyInventory_{i,q-1,y-2}}{AnnualInventory_{i,y-2}} + \frac{QuarterlyInventory_{i,q-1,y-3}}{AnnualInventory_{i,y-3}} \right) /2, \qquad (14)$$

where $QuarterlyInventory_{i,q,y}$ is firm i's inventory at the end of quarter q in fiscal year y, and *Annual-Inventory*$_{i,y}$ is its average inventory over the quarters in that year. Intuitively, we can view a high (low) $QInventory_{i,q,y}$ value as signalling that a firm held an abnormally high (low) inventory in the fiscal quarter prior to the current over the two fiscal years ending directly before the prior.

Combining the information in $QSales$, *Seasonality*, and $QInventory$, we can distinguish between seasonal inventory builders and non-inventory builders within and outside their high sales quarters. In particular, we can classify an observation with high values for all those variables as a seasonal firm within its high sales quarter which built up inventories to be sold within that quarter ("inventory builder"). Conversely, we can classify an observation with high values for $QSales$ and *Seasonality* but not $QInventory$ as a seasonal firm within its high sales quarter which did not build up inventories to be sold within that quarter ("instantaneous seller"). To more parsimoniously distinguish between those types of firms, we finally also define the dummy variable *DummyInventoryBuilder* to identify inventory builders. We set that dummy variable equal to one if $QSales$ and $QInventory$ attain their maximum value in the same fiscal quarter over the current fiscal year and else zero.

In our portfolio sorts, we control for Hou et al.'s (2015) q-theory, Fama and French's (2015) five-factor model, and Fama and French's (2018) six-factor model factors. Conversely, we add *MarketBeta*, *MarketSize*, *BookToMarket*, *Momentum*, *Investment*, and *Profitability* as control variables in our single-stock FM regressions. We calculate *MarketBeta* from Lewellen and Nagel (2006) regressions estimated over the prior twelve months of daily data. We define *MarketSize* as the log of the product of stock price and common shares outstanding at the end of the prior calendar year. *BookToMarket* is the log of the ratio of the book value of equity from the fiscal year end in the prior calendar year to the market value of equity at the end of the prior calendar year; *Momentum* is the log gross past return compounded over months $t-11$ to $t-1$; and *Investment* is the log gross percentage change in total assets over the fiscal year ending in the prior calendar year. Finally, *Profitability* is the ratio of sales minus costs of goods sold, selling, general, and administrative expenses, and interest expenses to the book value of equity, where all variables are from the fiscal year end in the prior calendar year.

We obtain market data from CRSP, accounting data from Compustat, and factor model data from Ken French and Lu Zhang. We study common stocks traded on the NYSE, Amex, or NASDAQ, excluding financial and utility stocks. We replace a stock's return with its delisting return whenever the delisting return is available. We exclude observations for which quarterly sales and/or quarterly inventory holdings are negative as well as those for which those variables are not available for the entire fiscal year. We only include observations for which the sum of quarterly sales is within a 5% bound of annual sales. In our FM regressions, we further exclude stocks with a price below $2 at the start of each sample month. With the exception of the stock return, we winsorize all variables at the 0.5th and 99.5th percentiles per month. Due to the availability of quarterly inventory data in Compustat, our sample period ranges from January 1979 to September 2019.

In Table 1, we offer descriptive statistics on *QSales*, *Seasonality*, *QInventory*, and *DummyInventory-Builder* (Panel A) as well as Spearman rank correlations for the set of those variables plus our control variables (Panel B). The descriptive statistics are the mean, standard deviation, skewness, kurtosis, several percentiles, and the number of observations. Except for the number of observations, we calculate the statistics in both panels first by sample month and then average over our sample period. Panel A reveals that the data contain 1,080,583 firm-month observations (including 9,531 unique firms). While the *QSales* and *QInventory* means are close to 0.25 by construction, the *Seasonality* mean suggests that the average firm observes a 0.03 standard deviation in *QSales* over an average fiscal year, while the *DummyInventoryBuilder* mean indicates that we classify about 36% of our sample firms as inventory builders. Panel B reports that the correlation between *QSales* and *QInventory* is 0.20, suggesting that firms display some tendency to build up inventories before their high sales season. Interestingly, the correlations between our main variables and the control variables are all close to zero. The only exceptions are the negative correlations of *DummyInventoryBuilder* with *MarketSize* (–0.18) and *Profitability* (–0.17), perhaps arising due to the fact that larger firms operating in more businesses tend to diversify away seasonalities in the single goods and/or services sold by them.

Table 1 about here.

3.2 The Seasonal Sales Premium

We next confirm that Grullon et al.'s (2020) finding that seasonal firms tend to have low (high) stock returns in their high (low) sales season also exists in our data. At the end of each sample month $t-1$, we thus first sort stocks into two portfolios according to the median of the *Seasonality* distribution in that month, referring to the high (low) *Seasonality* value stocks as seasonal (non-seasonal) stocks. Within each of these portfolios, we next sort stocks into three portfolios according to the tercile breakpoints of the *QSales* distribution for that portfolio and in that month. We refer to high (low) *QSales* stocks as stocks within (outside) their high sales quarter. We also form a spread portfolio long the highest *QSales* portfolio and short the lowest. We value-weight the portfolios and hold them over month t. To adjust portfolio returns for risk, we regress them on the q-theory, five-factor model, or six-factor model factors, reporting the intercepts ("alphas") from those regressions.

Table 2 gives the double-sorted portfolio results. While Panel A of the table reports mean portfolio returns in excess of the risk-free rate of return ("excess returns") and alphas, Panel B reports the mean number of stocks per portfolio as well as the time-series means of the cross-sectional means of several firm characteristics, all as plain numbers. The numbers in square brackets in Panel A are Newey and West (1987) t-statistics with a lag length of twelve months. The firm characteristics in Panel B include several of our control variables plus Heston and Sadka's (2008) *RSeason(xy)* and Chang et al.'s (2017) *ESeason*. *RSeason(xy)* is the same-calendar-month return averaged over the prior x calendar years, whereas *ESeason* is the average sales rank of the current fiscal quarter ($Q1$, $Q2$, $Q3$, or $Q4$) over the prior 20 fiscal quarters, with the ranking done in descending order. While columns (1), (2), (3), and (3)–(1) show the univariate *QSales* sorts conditional on high *Seasonality* stocks, columns (4), (5), (6), and (6)–(4) show those conditional on low *Seasonality* stocks.

Table 2 about here.

The portfolio sort results in the table strongly support Grullon et al.'s (2020) empirical evidence and the implications of our theoretical work. Panel A shows that the mean excess returns and alphas of seasonal stocks in columns (1) to (3) decline monotonically over the *QSales* portfolios, with the spreads in them over the portfolios always being highly significantly negative (see column (3)–(1)). While the mean excess return of seasonal stocks is, for example, 0.96% per month (t-statistic: 3.93) in their low sales quarter, it is a much lower 0.45% (t-statistic: 1.93) in their high sales quarter. The difference is a highly significant –0.50% (t-statistic: –3.64). In contrast, the mean excess returns and alphas of the non-seasonal stocks in columns (4) to (6) do not form discernible patterns over the *QSales* portfolios, leading the spreads in them over the portfolios to be economically and statistically insignificant (see column (6)–(4)). While the mean excess return of non-seasonal stocks is, for example, 0.69% per month (t-statistic: 3.47) in their low sales quarter, it is a close to identical 0.64% (t-statistic: 3.10) in their high sales quarter. The difference is an insignificant –0.05% (t-statistic: –0.59).

The firm characteristic statistics in Panel B suggest that seasonal firms tend to generate about 30% of their sales in their high sales quarter but only about 20% in their low sales quarter (compare *QSales* in columns (1) and (3)). Also, they show some weak tendency to build up output inventories to be sold in their high sales quarters (compare *QInventory* in those columns). In contrast, non-seasonal firms tend to generate about the same amount of sales in each quarter and do not tend to build up inventories to be sold in any quarter (see *QSales* and *QInventory* in columns (4) to (6)). Recognizing that firms with more seasonal sales are more likely to end up in the extreme *QSales* portfolios in columns (1) and (3) (or (4) and (6)), the firm characteristic statistics further suggest that seasonal stocks are, on average, smaller and less profitable than other stocks. Also, they tend to have similar book-to-market ratios, intermediate-term past returns, and investment expenditures relative to those other stocks. These findings are in agreement with the correlations in Table 1.

The final two rows of Panel B demonstrate that *ESeason*, which, just like *QSales*, also allows us to identify a firm's high and low earnings (and thus sales) quarters, rises over the *QSales* portfolios, indicating that the two variables broadly agree on the classification of quarters. They further show that *RSeason(xy)* decreases over those same portfolios, implying that seasonal firms do not only tend to produce low (high) stock returns in their current high (low) sales quarters but also in the same quarters over the prior three, five, and seven calendar years. Thus, the data do not only point to strong persistence in sales seasonality, but also in the accompanying stock return seasonality.

Overall, this section shows that seasonal firms tend to have low (high) stock returns in their high (low) sales quarters, in line with our theoretical conclusions. As this evidence has, however, already been reported in Grullon et al. (2020), we next study a novel implication of our theory.

3.3 Inventory Building and the Seasonal Sales Premium

We finally study the main novel implication of our theoretical work that a greater ability to build up inventories leads seasonal stocks to tend to have a lower (higher) stock return in their high (low) sales quarter. At the end of each sample month $t-1$, we thus again first sort stocks into two portfolios according to the median of the *Seasonality* distribution in that month. Within each of these portfolios, we next independently sort them into portfolios first according to the tercile breakpoints of *QSales* in that month and second according to the same breakpoints of *QInventory* in the same month. Within each *QSales* (*QInventory*) portfolio inside each *Seasonality* portfolio, we finally form a spread portfolio long the highest *QInventory* (*QSales*) portfolio and short the lowest. We value-weight the portfolios and hold them over month t. To adjust for risk, we again regress portfolio returns on the same sets of benchmark factors as before and report the intercepts ("alphas") from those regressions.

To interpret the triple-sorted portfolio results, we stress that a seasonal (non-seasonal) inventory builder ends up within the top *QSales*-top *QInventory* portfolio in the high (low) *Seasonality* portfolio within their high sales quarter but within the bottom *QSales*-bottom *QInventory* portfolio in the same *Seasonality* portfolio within their low sales quarter. As a result, we can measure the seasonal sales premium of seasonal (non-seasonal) inventory builders from the spread portfolio long the former triple-sorted portfolio and short the latter. In accordance, a seasonal (non-seasonal) non-inventory builder ends up within the top *QSales*-bottom *QInventory* portfolio in the high (low) *Seasonality* portfolio within their high sales quarter but within the bottom *QSales*-top *QInventory* portfolio in the same *Seasonality* portfolio within their low sales quarter. In the same spirit as before, we can thus measure the seasonal sales premium of seasonal (non-seasonal) non-inventory builders from the spread portfolio long the former triple-sorted portfolio and short the latter.

Table 3 offers the mean excess returns of the triple-sorted portfolios (Panel A), the time-series means of their cross-sectional *QSales* and *QInventory* means (Panels B and C, respectively), and their mean number of stocks (Panel D). As before, the numbers in square brackets in Panel A are Newey and West (1987) t-statistics with a twelve-month lag length. Also as before, columns (1), (2), (3), and (3)–(1) focus on the high *Seasonality* stocks, while columns (4), (5), (6), and (6)–(4) focus on the low

Seasonality stocks. Panel A strongly supports our theoretical conclusions, suggesting that seasonal inventory builders produce a significantly lower seasonal sales premium than seasonal non-inventory builders or non-seasonal stocks. While the mean excess return of seasonal inventory builders is an insignificant 0.30% per month (t-statistic: 1.39) in their high sales quarter, it is a significant 1.22% (t-statistic: 4.96) in their low sales quarter, yielding a seasonal sales premium of –0.91% (t-statistic: –5.82). In contrast, the mean excess returns of seasonal non-inventory builders are a more similar 0.32% and 0.59% (t-statistics: 0.97 and 1.57) in their high and low sales quarter, all respectively, yielding an insignificant seasonal sales premium of –0.27% (t-statistic: –0.83). In the same vein, non-seasonal inventory or non-inventory builders also fail to yield significant seasonal sales premiums.

TABLE 3 ABOUT HERE.

While Panel B confirms that seasonal and non-seasonal firms observe similar variations in *QSales* as in the two-way sorts in Table 2, Panel C reveals that the inventory builders among those firms observe close to equally large variations in *QInventory* as in *QSales*. Seasonal inventory builders, for example, do not only see their mean *QSales* values rise from 0.20 to 0.30 from their low to high sales quarters, but also their mean *QInventory* values from 0.21 to 0.29 over the same period. Panel D finally shows that all triple-sorted portfolios are well diversified in terms of stock numbers.

Table 4 reports the q-theory model (Panel A), five-factor model (Panel B), and six-factor model (Panel C) alphas of the triple-sorted portfolios. The design of each panel is identical to that of Panel A of Table 3. The table offers strong evidence that controlling for the benchmark factors of those models does not materially affect our conclusions. While the seasonal sales premium in the inventory builder subsample is –0.91% per month (t-statistic: –5.82) in the absence of controls (see again Panel A of Table 3), it is a close –0.86%, –0.91%, and –0.92% (t-statistics: –4.93, –5.48, and –5.50) controlling for the q-theory, five-factor model, and six-factor model factors, all respectively. In the same vein, the seasonal sales premiums in the seasonal non-inventory-builder and the non-seasonal inventory-builder and non-inventory-builder subsamples also stay similar to their previous values.[7]

[7] Our triple-sorted portfolio results are robust to reasonable variations in methodology. Specifically, conducting an entirely dependent triple-sort, the seasonal sales premium is –0.88% per month (t-statistic: –6.75) among seasonal inventory builders but only –0.28% (t-statistic: –1.06) among seasonal non-inventory builders. Moreover, exclusively using NYSE stocks to compute breakpoints in the original sorts, that premium is –0.69% (t-statistic: –4.30) among seasonal inventory builders but only 0.06% (t-statistic: 0.24) among seasonal non-inventory builders.

Table 4 about here.

We finally conduct FM regressions to estimate the conditional effect of inventory building on the seasonal sales premium, assessing the robustness of our conclusions to variations in the methodology used. In the regressions, we project the excess returns of single stocks over month t on a monotonic transformation of *QSales* and the controls measured at the end of month $t-1$. We run the regressions separately on the full samples of seasonal and non-seasonal stocks as well as the subsamples of seasonal and non-seasonal inventory builders and non-inventory builders. While seasonal (non-seasonal) stocks have a *Seasonality* value above (below) the median at the end of month $t-1$, inventory builders (non-inventory builders) have a *DummyInventoryBuilder* value equal to one (zero) at the end of that month. To mitigate outlier effects and to ensure that results can be compared across subsamples, we employ the rank of *QSales* ("*QSalesRank*") rather than *QSales* in the regressions.[8] To alleviate microstructure issues, we also omit stocks with a below \$2 price at the end of month $t-1$.

Table 5 presents the FM regression results. While columns (1) to (3) focus on the full sample of seasonal stocks, seasonal inventory builders, and seasonal non-inventory builders, columns (4) to (6) focus on their non-seasonal counterparts, all respectively. Conversely, column (2)–(3) ((5)–(6)) shows the differences in estimates between the seasonal (non-seasonal) inventory and non-inventory builders. Plain numbers are monthly premium estimates, whereas the numbers in square parentheses are Newey and West (1987) t-statistics with a twelve-month lag length. The regressions yield results in line with the portfolio sorts. In particular, columns (1) and (4) suggest that seasonal — but not non-seasonal — stocks produce a significant seasonal sales premium. More crucially, while seasonal inventory builders in column (2) yield a significantly negative *QSalesRank* premium of –0.62% per month (t-statistic: –4.25), the same premium is a much less significant –0.20% (t-statistic: –1.79) for seasonal non-inventory builders in column (3). The difference is a significant –0.40% (t-statistic: –2.45). Conversely, while non-seasonal inventory builders in column (5) yield a significantly negative *QSalesRank* premium of –0.34% (t-statistic: –3.09), the same premium is an insignificant –0.02%

[8] We obtain similar results from subsample FM regressions on *QSales* and the controls. While the seasonal inventory builder subsample, for example, yields a *QSales* premium of –3.79% per month (t-statistic: –4.07), the same subsample yields a *QSalesRank* premium of –0.63% (t-statistic: –4.25). We also obtain similar results from FM regressions jointly run on seasonal or non-seasonal stocks and featuring *DummyInventoryBuilder* and an interaction between *QSales* and *DummyInventoryBuilder*. The seasonal-firm regression, for example, yields an insignificant *QSales* premium of –1.11% per month (t-statistic: –1.47) but a highly significant interaction premium of –2.51% (t-statistic: –2.33).

(t-statistic: –0.24) for non-seasonal non-inventory builders in column (6). The difference is a significant –0.33% (t-statistic: –2.81). The controls yield premiums in line with the literature.

Table 5 about here.

Taken together, this section shows that a greater ability to build up inventories yields a significantly more negative seasonal sales premium, in complete accordance with our theory. While prior studies offer alternative explanations for that premium based on investor mood or inattention (see, e.g., Grullon et al. (2020), Hirshleifer et al. (2020), and Keloharju et al. (2021), etc.), it is unclear how those could account for the conditioning effect of inventory building shown in this section.

4 Robustness Tests

In this section, we offer robustness test results. We first establish that conditioning the seasonal sales premium on an inventory building proxy calculated from output — and not total — inventories does not change our conclusions. We next show that excluding January months from our tests, as also done by Heston and Sadka (2008) and Keloharju et al. (2016), neither changes those conclusions.

4.1 Using Output Inventories to Measure Inventory Building

Although our theory analyzes how a firm's ability to build up *output* inventories shapes the relation between seasonalities in its output price (or: sales) and expected return, we nonetheless use quarterly *total* inventories (Compustat item: invtq) also including raw material and work-in-progress to measure inventory building in our empirical work. We do so since, while quarterly total inventories data are available in Compustat from 1979, quarterly finished goods inventories data (Compustat item: invfgq) are available only from 2008 for a meaningful number of firms. Given, however, that quarterly finished goods inventories make up the lion share of quarterly total goods inventories, and that the average cross-sectional correlation between those two inventory variables (scaled by quarterly total assets) is 0.78, it is unlikely that using quarterly total inventories greatly distorts our findings.

Notwithstanding, Table 6 repeats our triple portfolio sorts in Tables 3 and 4 using an alternative version of *QInventory*, *QFGInventory*, computed analogous to *QInventory* except that we use quarterly finished goods (and not total goods) inventories data in its computation. To be concise, the table only reports the mean excess returns and alphas of spread portfolios long the top *QSales*-top *QFGInventory* portfolio and short the bottom *QSales*-bottom *QFGInventory* portfolio ("inventory builder") or long the top *QSales*-bottom *QFGInventory* portfolio and short the bottom *QSales*-top *QFGInventory* portfolio ("non-inventory builder") formed using seasonal (column (1)) or non-seasonal (column (2)) stocks. While Panel A presents the mean excess returns of the spread portfolios, Panels B, C, and D present their q-theory, five-factor model, and six-factor model alphas, respectively.

TABLE 6 ABOUT HERE.

Despite the much shorter sample period from July 2008 to April 2019, the triple portfolio sorts based on *QFGInventory* yield results in agreement with those based on *QInventory*. While column (1) in Panel A, for example, suggests that seasonal inventory builders yield a seasonal sales premium of –1.50% per month (t-statistic: –3.62), the corresponding premium for non-inventory builders is an insignificant 0.64% (t-statistic: 1.06). Also as before, column (2) in that panel shows that non-seasonal inventory or non-inventory builders again only yield insignificant seasonal sales premiums. Adjusting the mean excess returns for risk does again not change our conclusions (see Panels B to D).

4.2 Excluding January Months

Since the high sales season of a large number of firms is the Christmas season at the end of the calendar year, and as stock returns tend to be higher in January (see, e.g., Rozeff and Kinney (1976) and Keim (1983)), it is conceivable that the seasonal sales premium is a manifestation of the higher January returns. To rule out that possibility, Table 7 follows Heston and Sadka (2008) and Keloharju et al. (2016) in repeating our triple portfolio sorts in Tables 3 and 4 excluding January months. The design of the table is identical to that of Table 6. Refuting the idea that January returns drive our findings, the table suggests that the non-January months produce conclusions in agreement with our main conclusions. While Panel A shows that the seasonal inventory builders yield a seasonal sales premium of –1.07% per month (t-statistic: –6.64) outside of January, the corresponding premium for seasonal non-inventory builders is an insignificant –0.06% (t-statistic: –0.44) then. The same table further shows that the non-seasonal firms continue to produce insignificant seasonal sales premiums, while Panel B to D again suggest that adjusting for risk does not materially alter our results.

Table 7 about here.

5 Conclusion

We consider a real options asset pricing model featuring a firm exposed to seasonal variations in its output price and able to build up output inventories to be sold later. The model yields the prediction that the seasonality in the output price only translates into negatively-correlated seasonality in the expected return if firms find it cheap to build up output inventories and thus start doing so long before their high price season. Using quarterly sales and inventory data, we offer strong empirical support for that prediction, showing that seasonal firms only produce low (high) mean stock returns in their high (low) sales quarter if they hold abnormally high (low) amounts of inventories at the start of that quarter. Our conclusions are important since they suggest that seasonalities in stock returns unveiled in recent research are consistent with neoclassical finance theory.

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Table 1
Descriptive Statistics
The table presents descriptive statistics for *QSales*, *Seasonality*, *QInventory*, and *DummyInventoryBuilder* (Panel A) and Spearman rank correlations between the set of those variables and controls (Panel B). We calculate both the descriptive statistics and the correlations by sample month and then average over our sample period. All variables are winsorized at the 0.5th and 99.5th percentiles computed per month. The sample period is January 1979 to September 2019. See Table C1 in Appendix C for variable definitions.

	QSales	*Seasonality*	*QInventory*	*Dummy Inventory Builder*
	(1.00)	(2.00)	(3.00)	(4.00)
		Panel A: Descriptive Statistics		
Mean	0.25	0.03	0.25	0.36
Standard Deviation	0.04	0.03	0.03	0.48
Skewness	0.52	2.67	0.35	0.58
Excess Kurtosis	5.23	9.59	7.73	−1.65
Percentile 1	0.15	0.00	0.14	0.00
Percentile 5	0.20	0.01	0.20	0.00
Quartile 1	0.23	0.01	0.24	0.00
Median	0.25	0.02	0.25	0.00
Quartile 3	0.26	0.04	0.26	1.00
Percentile 95	0.31	0.09	0.30	1.00
Percentile 99	0.37	0.17	0.36	1.00
Observations	1,080,583	1,080,583	1,080,583	1,080,583
		Panel B: Spearman Rank Correlations		
Seasonality	−0.06			
QInventory	0.20	−0.03		
DummyInventoryBuilder	−0.02	0.15	−0.03	
MarketBeta	0.00	0.05	0.00	−0.01
MarketSize	0.00	−0.18	0.00	0.00
BookToMarket	0.02	−0.04	0.02	0.01
Momentum	0.00	−0.07	0.01	0.00
Investment	−0.03	0.08	−0.07	0.01
Profitability	0.00	−0.17	−0.02	0.02

Table 2
Univariate *QSales* Sorts
The table presents the mean excess returns and alphas of portfolios sorted on *QSales* (Panel A) as well as characteristics of those portfolios (Panel B). At the end of each sample month $t-1$, we first sort stocks into portfolios based on the median of the *Seasonality* distribution at that time. Within each *Seasonality* portfolio, we next form three portfolios based on the 33rd and 66th percentiles of the *QSales* distribution at that time. We value-weight the portfolios and hold them over month t. We also form spread portfolios long the highest *QSales* portfolio and short the lowest within each *Seasonality* portfolio ("High–Low"). The plain numbers in Panel A are monthly mean excess returns (Return^e) and the q-theory model (q), Fama-French (2015) five-factor model (FF5), and Fama-French (2018) six-factor model (FF6) alphas in percent, while the numbers in square brackets are Newey and West (1987) t-statistics with a twelve-month lag length. The plain numbers in Panel B are the mean number of stocks (# Stocks) and the time-series means taken over the cross-sectional means of several well-known firm characteristics. See Table C1 in Appendix C for variable definitions.

	Seasonality							
	High (Above Median)				Low (Below Median)			
	QSales				*QSales*			
	Low	Med.	High	H–L	Low	Med.	High	H–L
	(1.00)	(2.00)	(3.00)	(3.00)–(1)	(4.00)	(5.00)	(6.00)	(6.00)–(4)
Panel A: Portfolio Returns and Alphas								
Return^e	0.96	0.60	0.45	−0.50	0.69	0.84	0.64	−0.05
	[3.93]	[2.75]	[1.93]	[−3.64]	[3.47]	[4.68]	[3.10]	[−0.59]
q Alpha	0.43	−0.04	−0.12	−0.55	−0.09	0.10	−0.07	0.01
	[3.16]	[−0.50]	[−1.29]	[−3.46]	[−1.27]	[1.44]	[−0.99]	[0.13]
FF5 Alpha	0.32	−0.13	−0.20	−0.52	−0.11	0.07	−0.12	−0.01
	[3.07]	[−1.59]	[−2.35]	[−3.61]	[−1.68]	[1.17]	[−1.58]	[−0.11]
FF6 Alpha	0.37	−0.09	−0.17	−0.54	−0.11	0.05	−0.10	0.01
	[3.35]	[−1.09]	[−1.96]	[−3.68]	[−1.62]	[0.92]	[−1.30]	[0.07]
Panel B: Portfolio Characteristics								
# Stocks	338.60	371.15	361.27	22.67	382.52	359.67	396.48	13.96
QSales	0.20	0.25	0.30	0.10	0.24	0.25	0.26	0.03
QInventory	0.24	0.25	0.26	0.02	0.25	0.25	0.25	0.00
MarketBeta	1.09	1.09	1.09	0.00	1.02	1.00	1.02	0.00
Market Size	4.82	5.24	4.84	0.02	5.55	5.78	5.49	−0.05
BookToMarket	−0.64	−0.63	−0.60	0.04	−0.55	−0.54	−0.52	0.04
Momentum	−0.02	0.00	−0.02	0.00	0.05	0.06	0.04	0.00
Investment	0.13	0.14	0.14	0.01	0.07	0.07	0.07	0.00
Profitability	0.03	0.14	0.05	0.02	0.22	0.24	0.22	0.00
RSeason (3y)	2.30	1.82	1.80	−0.50	1.56	1.37	1.44	−0.12
RSeason (5y)	2.03	1.74	1.65	−0.39	1.57	1.33	1.46	−0.11
RSeason (7y)	1.82	1.59	1.50	−0.33	1.51	1.31	1.39	−0.12
ESeason	8.54	10.31	12.45	3.91	9.32	10.42	11.54	2.22

Table 3

Double-Sorted *QSales* and *QInventory* Portfolios

The table presents the mean excess returns of portfolios independently-sorted based on *QSales* and *QInventory* (Panel A), the mean *QSales* and *QInventory* values of those portfolios (Panels B and C, respectively), and the mean number of stocks per portfolio (Panel D). At the end of each sample month $t-1$, we first sort stocks into portfolios based on the median of the *Seasonality* distribution at that time. Within each *Seasonality* portfolio, we next independently sort stocks into portfolios based on, first, the 33rd and 66th percentiles of the *QSales* distribution at that time and, second, based on the same percentiles of the *QInventory* distribution at that time. We value-weight the portfolios and hold them over month t. We also form spread portfolios long the highest *QSales* (*QInventory*) portfolio and short the lowest within each *QInventory* (*QSales*) portfolio ("High–Low") per *Seasonality* portfolio. In addition, we also create spread portfolios long the top *QSales*/top *QInventory* portfolio and short the bottom *QSales*/bottom *QInventory* portfolio ("inventory builders") and long the top *QSales*/bottom *QInventory* portfolio and short the bottom *QSales*/top *QInventory* portfolio ("non-inventory builders") per *Seasonality* portfolio. The plain numbers in Panel A are monthly mean excess returns in percent, while the numbers in square brackets are Newey and West (1987) t-statistics with a twelve-month lag length. The plain numbers in Panels B to C are the time-series means taken over the cross-sectional means of *QSales* and *QInventory*, respectively. The plain numbers in Panel D are the average number of stocks. See Table C1 in Appendix C for variable definitions.

	Seasonality							
	High (Above Median)				Low (Below Median)			
	QSales				*QSales*			
QInventory	Low	Med.	High	H–L	Low	Med.	High	H–L
	(1)	(2)	(3)	(3)–(1)	(4)	(5)	(6)	(6)–(4)
Panel A: Value-Weighted Portfolio Returns								
Low	1.22	0.75	0.32	−0.90	0.56	0.96	0.70	0.14
	[4.96]	[3.13]	[0.97]	[−4.19]	[2.40]	[4.66]	[2.60]	[0.94]
Medium	0.78	0.62	0.57	−0.21	0.90	0.82	0.70	−0.20
	[3.18]	[2.59]	[2.21]	[−1.28]	[5.01]	[4.52]	[3.47]	[−1.76]
High	0.59	0.45	0.30	−0.29	0.65	0.67	0.56	−0.09
	[1.57]	[1.98]	[1.39]	[−0.88]	[2.78]	[3.18]	[2.60]	[−0.65]
High–Low	−0.63	−0.30	−0.01		0.09	−0.29	−0.14	
	[−1.94]	[−1.80]	[−0.06]		[0.78]	[−2.10]	[−0.75]	
Seasonal Sales Premium$_{\text{Inventory Builder}}$				−0.91				0.00
($QSales(3)\&QInventory(3)$ – $QSales(1)\&QInventory(1)$)				[−5.82]				[0.01]
Seasonal Sales Premium$_{\text{Non-Inventory Builder}}$				−0.27				0.05
($QSales(3)\&QInventory(1)$ – $QSales(1)\&QInventory(3)$)				[−0.83]				[0.28]
Panel B: Mean *QSales*								
Low	0.20	0.25	0.30	0.10	0.24	0.25	0.26	0.03
Medium	0.21	0.25	0.29	0.09	0.24	0.25	0.26	0.03
High	0.20	0.25	0.31	0.11	0.24	0.25	0.26	0.03
High–Low	0.00	0.00	0.01		0.00	0.00	0.00	

(continued on next page)

Table 3
Double-Sorted *QSales* and *QInventory* Portfolios (cont.)

	Seasonality							
	High (Above Median)				Low (Below Median)			
	QSales				*QSales*			
QInventory	Low	Med.	High	H–L	Low	Med.	High	H–L
	(1)	(2)	(3)	(3)–(1)	(4)	(5)	(6)	(6)–(4)
Panel C: Mean *QInventory*								
Low	0.21	0.22	0.21	0.00	0.23	0.23	0.23	0.00
Medium	0.25	0.25	0.25	0.00	0.25	0.25	0.25	0.00
High	0.29	0.28	0.29	−0.01	0.27	0.27	0.27	0.00
High–Low	0.08	0.07	0.08		0.05	0.04	0.04	
Panel D: Mean Number of Stocks in Portfolio								
Low	148.88	123.68	77.49		135.09	111.13	100.11	
Medium	102.32	148.54	115.35		132.19	144.71	148.84	
High	87.46	98.98	168.49		115.28	103.86	147.58	

Table 4
Risk-Adjusted Double-Sorted *QSales* and *QInventory* Portfolios
The table presents the q-theory (Panel A), five-factor (Panel B), and six-factor (Panel C) model alphas of portfolios independently-sorted based on *QSales* and *QInventory*. At the end of each sample month $t-1$, we first sort stocks into portfolios based on the median of the *Seasonality* distribution at that time. Within each *Seasonality* portfolio, we next independently sort stocks into portfolios based on, first, the 33rd and 66th percentiles of the *QSales* distribution at that time and, second, based on the same percentiles of the *QInventory* distribution at that time. We value-weight the portfolios and hold them over month t. We also form spread portfolios long the highest *QSales* (*QInventory*) portfolio and short the lowest within each *QInventory* (*QSales*) portfolio ("High–Low") per *Seasonality* portfolio. In addition, we also create spread portfolios long the top *QSales*/top *QInventory* portfolio and short the bottom *QSales*/bottom *QInventory* portfolio ("inventory builders") and long the top *QSales*/bottom *QInventory* portfolio and short the bottom *QSales*/top *QInventory* portfolio ("non-inventory builders") per *Seasonality* portfolio. The plain numbers in each panel are the constants from time-series regressions of the portfolio excess return on the appropriate benchmark factors in percent, while the numbers in square brackets are Newey and West (1987) t-statistics with a twelve-month lag length. See Table C1 in Appendix C for variable definitions.

	Seasonality							
	High (Above Median)				Low (Below Median)			
	QSales				*QSales*			
QInventory	Low	Med.	High	H–L	Low	Med.	High	H–L
	(1)	(2)	(3)	(3)–(1)	(4)	(5)	(6)	(6)–(4)
Panel A: q-Theory Model Alphas								
Low	0.57	0.16	−0.16	−0.72	−0.17	0.23	0.19	0.36
	[3.35]	[1.05]	[−0.96]	[−3.12]	[−1.82]	[1.59]	[1.15]	[1.91]
Medium	0.11	−0.08	−0.06	−0.17	0.11	0.02	−0.12	−0.23
	[0.85]	[−0.64]	[−0.46]	[−1.02]	[1.23]	[0.25]	[−1.28]	[−1.75]
High	0.23	−0.18	−0.29	−0.52	−0.22	0.03	−0.16	0.06
	[0.60]	[−1.27]	[−1.98]	[−1.18]	[−1.91]	[0.36]	[−1.51]	[0.36]
High–Low	−0.34	−0.34	−0.13		−0.05	−0.20	−0.35	
	[−0.77]	[−1.57]	[−0.70]		[−0.36]	[−1.25]	[−1.78]	
Seasonal Sales Premium$_{\text{Inventory Builder}}$				−0.86				0.01
($QSales(3)\&QInventory(3) - QSales(1)\&QInventory(1)$)				[−4.93]				[0.08]
Seasonal Sales Premium$_{\text{Non-Inventory Builder}}$				−0.38				0.41
($QSales(3)\&QInventory(1) - QSales(1)\&QInventory(3)$)				[−0.90]				[1.64]

(continued on next page)

Table 4
Risk-Adjusted Double-Sorted *QSales* and *QInventory* Portfolios (cont.)

	Seasonality							
	High (Above Median)				Low (Below Median)			
	QSales				*QSales*			
QInventory	Low	Med.	High	H–L	Low	Med.	High	H–L
	(1)	(2)	(3)	(3)–(1)	(4)	(5)	(6)	(6)–(4)
Panel B: Fama-French 5-Factor Model Alphas								
Low	0.50	0.04	−0.22	−0.72	−0.18	0.22	0.08	0.26
	[3.49]	[0.30]	[−1.41]	[−3.37]	[−2.02]	[1.67]	[0.53]	[1.56]
Medium	0.03	−0.13	−0.16	−0.19	0.08	0.00	−0.16	−0.24
	[0.24]	[−1.08]	[−1.23]	[−1.13]	[0.96]	[−0.02]	[−1.81]	[−1.96]
High	0.07	−0.28	−0.41	−0.49	−0.24	−0.03	−0.22	0.01
	[0.24]	[−1.97]	[−3.36]	[−1.31]	[−2.31]	[−0.33]	[−1.96]	[0.08]
High–Low	−0.43	−0.32	−0.19		−0.06	−0.25	−0.30	
	[−1.09]	[−1.61]	[−1.06]		[−0.49]	[−1.59]	[−1.58]	
Seasonal Sales Premium$_{\text{Inventory Builder}}$				−0.91				−0.04
(*QSales*(3)&*QInventory*(3) − *QSales*(1)&*QInventory*(1))				[−5.48]				[−0.29]
Seasonal Sales Premium$_{\text{Non-Inventory Builder}}$				−0.30				0.31
(*QSales*(3)&*QInventory*(1) − *QSales*(1)&*QInventory*(3))				[−0.80]				[1.45]
Panel C: Fama-French 6-Factor Model Alphas								
Low	0.57	0.10	−0.13	−0.70	−0.18	0.19	0.11	0.28
	[4.13]	[0.72]	[−0.85]	[−3.10]	[−1.90]	[1.58]	[0.65]	[1.51]
Medium	0.09	−0.09	−0.14	−0.23	0.08	−0.01	−0.13	−0.21
	[0.66]	[−0.77]	[−1.07]	[−1.36]	[0.87]	[−0.16]	[−1.57]	[−1.67]
High	0.06	−0.25	−0.35	−0.41	−0.22	−0.03	−0.20	0.02
	[0.21]	[−1.84]	[−3.04]	[−1.20]	[−2.09]	[−0.28]	[−1.77]	[0.13]
High–Low	−0.50	−0.35	−0.22		−0.05	−0.22	−0.30	
	[−1.41]	[−1.73]	[−1.24]		[−0.37]	[−1.50]	[−1.57]	
Seasonal Sales Premium$_{\text{Inventory Builder}}$				−0.92				−0.02
(*QSales*(3)&*QInventory*(3) − *QSales*(1)&*QInventory*(1))				[−5.50]				[−0.15]
Seasonal Sales Premium$_{\text{Non-Inventory Builder}}$				−0.19				0.33
(*QSales*(3)&*QInventory*(1) − *QSales*(1)&*QInventory*(3))				[−0.56]				[1.37]

Table 5
Fama-MacBeth Regressions of Stock Returns on *QSales*
The table presents the results from Fama-MacBeth (1973) regressions of excess stock returns over month t on pricing variables measured until the start of that month. In columns (1), (2), and (3), we run the regressions on stocks with a *Seasonality* value above the median, whereas in columns (4), (5), and (6) we run them on stocks with a value below the median. Conversely, in each set of columns, we run the regressions on all firms (columns (1) and (4)), those with a *DummyInventoryBuilder* value equal to one ("inventory builders;" columns (2) and (5)), and those with a *DummyInventoryBuilder* value equal to zero ("non-inventory builders;" columns (3) and (6)). We also report the differences in outcomes across the subsample estimations (columns (2)–(3) and (5)–(6)). Plain numbers are risk premiums or differences in those, by month and in percent. The numbers in square brackets are Newey and West (1987) t-statistics with a lag length of twelve months. We exclude stocks whose price is below \$2 at the start of month t. See Table C1 in Appendix C for variable definitions.

	Fama-MacBeth Regressions							
	Seasonality Above Median				*Seasonality* Below Median			
	All Firms	Inv. Builder	Non-Inv. Builder	Diff.	All Firms	Inv. Builder	Non-Inv. Builder	Diff.
	(1)	(2)	(3)	(2)–(3)	(4)	(5)	(6)	(5)–(6)
QSalesRank	−0.43	−0.63	−0.20	−0.40	−0.11	−0.34	−0.02	−0.33
	[−4.18]	[−4.25]	[−1.79]	[−2.45]	[−1.41]	[−3.09]	[−0.24]	[−2.81]
MarketBeta	0.16	0.23	0.12	0.06	0.15	0.13	0.16	−0.11
	[0.99]	[1.22]	[0.75]	[0.65]	[0.84]	[0.69]	[0.95]	[−1.36]
MarketSize	−0.01	0.01	−0.03	0.05	−0.04	−0.02	−0.05	0.04
	[−0.41]	[0.27]	[−0.81]	[1.92]	[−1.11]	[−0.46]	[−1.46]	[1.85]
BookToMarket	0.23	0.24	0.26	0.03	0.17	0.21	0.15	0.06
	[3.22]	[2.92]	[3.37]	[0.49]	[2.37]	[2.53]	[2.11]	[0.93]
Momentum	1.04	1.22	0.90	1.22	0.56	0.69	0.50	0.69
	[6.07]	[6.80]	[4.64]	[6.77]	[2.79]	[2.65]	[2.63]	[2.63]
Investment	−0.40	−0.67	−0.20	−0.37	−0.19	−0.61	0.00	−0.62
	[−3.49]	[−4.55]	[−1.34]	[−1.96]	[−1.64]	[−2.97]	[0.03]	[−2.49]
Profitability	0.35	0.55	0.32	0.20	0.45	0.35	0.56	−0.24
	[3.20]	[3.72]	[2.67]	[1.41]	[3.45]	[1.96]	[3.27]	[−1.09]
Constant	0.73	0.62	0.75	−0.26	0.83	0.86	0.82	−0.01
	[2.19]	[1.80]	[2.23]	[−1.50]	[2.52]	[2.63]	[2.36]	[−0.09]

Table 6

Double-Sorted *QSales* and *QInventory* Portfolios Based On Finished-Goods Inventories

The table presents the mean excess returns (Panel A) as well as the q-theory (Panel B), five-factor (Panel C), and six-factor (Panel D) model alphas of independently-sorted portfolios based on *QSales* and *QFGInventory*. At the end of each sample month $t-1$, we first sort stocks into portfolios based on the median of the *Seasonality* distribution at that time. Within each *Seasonality* portfolio, we next independently sort stocks into portfolios based on, first, the 33rd and 66th percentiles of the *QSales* distribution at that time and, second, based on the same percentiles of the *QInventory* distribution at that time. We value-weight the portfolios and hold them over month t. We next create spread portfolios long the top *QSales*/top *QInventory* portfolio and short the bottom *QSales*/bottom *QInventory* portfolio ("inventory builders") and long the top *QSales*/bottom *QInventory* portfolio and short the bottom *QSales*/top *QInventory* portfolio ("non-inventory builders") per *Seasonality* portfolio. Plain numbers are monthly mean excess returns in percent, while the numbers in square brackets are Newey and West (1987) t-statistics with a twelve-month lag length. See Table C1 in Appendix C for variable definitions.

	Seasonality	
	Above Median	Below Median
	(1.00)	(2.00)
Panel A: Value-Weighted Portfolio Returns		
Seasonal Sales Premium$_{\text{Inventory Builder}}$	−1.50	0.12
	[−3.62]	[0.39]
Seasonal Sales Premium$_{\text{Non-Inventory Builder}}$	0.64	−0.23
	[1.06]	[−0.69]
Panel B: q-Theory Model Alphas		
Seasonal Sales Premium$_{\text{Inventory Builder}}$	−1.61	0.18
	[−3.23]	[0.52]
Seasonal Sales Premium$_{\text{Non-Inventory Builder}}$	0.82	−0.28
	[1.10]	[−0.73]
Panel C: Fama-French 5-Factor Model Alphas		
Seasonal Sales Premium$_{\text{Inventory Builder}}$	−1.78	0.03
	[−3.68]	[0.08]
Seasonal Sales Premium$_{\text{Non-Inventory Builder}}$	1.22	−0.44
	[1.80]	[−1.06]
Panel D: Fama-French 6-Factor Model Alphas		
Seasonal Sales Premium$_{\text{Inventory Builder}}$	−1.79	0.01
	[−3.74]	[0.04]
Seasonal Sales Premium$_{\text{Non-Inventory Builder}}$	1.22	−0.46
	[1.76]	[−1.09]

Table 7
Double-Sorted *QSales* and *QInventory* Portfolios Excluding January Observations
The table presents the mean excess returns (Panel A) as well as the q-theory (Panel B), five-factor (Panel C), and six-factor (Panel D) model alphas of independently-sorted portfolios based on *QSales* and *QInventory* derived from our sample data excluding January observations. At the end of each sample month $t-1$, we first sort stocks into portfolios based on the median of the *Seasonality* distribution at that time. Within each *Seasonality* portfolio, we next independently sort stocks into portfolios based on, first, the 33rd and 66th percentiles of the *QSales* distribution at that time and, second, based on the same percentiles of the *QInventory* distribution at that time. We value-weight the portfolios and hold them over month t. We next create spread portfolios long the top *QSales*/top *QInventory* portfolio and short the bottom *QSales*/bottom *QInventory* portfolio ("inventory builders") and long the top *QSales*/bottom *QInventory* portfolio and short the bottom *QSales*/top *QInventory* portfolio ("non-inventory builders") per *Seasonality* portfolio. Plain numbers are monthly mean excess returns in percent, while the numbers in square brackets are Newey and West (1987) t-statistics with a twelve-month lag length. See Table C1 in Appendix C for variable definitions.

	Seasonality	
	Above Median	Below Median
	(1.00)	(2.00)
Panel A: Value-Weighted Portfolio Returns		
Seasonal Sales Premium$_{\text{Inventory Builder}}$	−1.07	−0.06
	[−6.64]	[−0.44]
Seasonal Sales Premium$_{\text{Non-Inventory Builder}}$	−0.32	−0.01
	[−0.99]	[−0.04]
Panel B: q-Theory Model Alphas		
Seasonal Sales Premium$_{\text{Inventory Builder}}$	−1.11	−0.07
	[−5.64]	[−0.50]
Seasonal Sales Premium$_{\text{Non-Inventory Builder}}$	−0.53	0.34
	[−1.16]	[1.25]
Panel C: Fama-French 5-Factor Model Alphas		
Seasonal Sales Premium$_{\text{Inventory Builder}}$	−1.11	−0.11
	[−6.23]	[−0.87]
Seasonal Sales Premium$_{\text{Non-Inventory Builder}}$	−0.40	0.26
	[−1.04]	[1.16]
Panel D: Fama-French 6-Factor Model Alphas		
Seasonal Sales Premium$_{\text{Inventory Builder}}$	−1.17	−0.12
	[−6.32]	[−0.91]
Seasonal Sales Premium$_{\text{Non-Inventory Builder}}$	−0.29	0.24
	[−0.84]	[1.00]

A Model Solution

In this appendix, we offer details on how we numerically find the value and the expected return of the firm in the model. We start with detailing the firm's optimal production and selling decisions. We then continue with introducing our finite difference scheme. We finally derive the boundary conditions for our finite difference scheme and discuss how we "knit together" the separate solution components.

A.1 The Firm's Optimal Policies

Recall that the differential of the output price P_t in our main model obeys

$$\mathrm{d}P_t = (\alpha + \kappa \sin(\eta t))P_t \mathrm{d}t + \sigma P_t \mathrm{d}B_t, \tag{A1}$$

and let μ be the expected return of a portfolio replicating the stochastic variations in the output price and thus reflecting its systematic risk. The "expected-return shortfall" of the output price can then be written as $\delta_t = \mu - \frac{1}{P_t \mathrm{d}t}\mathbb{E}[\mathrm{d}P_t] = \mu - \alpha - \kappa \sin(\eta t)$, which is a sinusoidal function of time. Using the expected-return shortfall δ_t, we can rewrite the output price differential in Equation (A1) as

$$\mathrm{d}P_t = (\mu - \delta_t)P_t \mathrm{d}t + \sigma P_t \mathrm{d}B_t, \tag{A2}$$

whose closed-form solution is well known to be equal to

$$\begin{aligned} P_t &= P_0 \exp\left(\int_0^t (\mu - \delta_u)\mathrm{d}u - \frac{1}{2}\sigma^2 t + \sigma B_t\right) && \text{(A3)} \\ &= P_0 \exp\left(\left(\alpha - \frac{1}{2}\sigma^2\right) t + \frac{\kappa}{\eta}\left(1 - \cos(\eta t)\right) + \sigma B_t\right). && \text{(A4)} \end{aligned}$$

Under the martingale measure, the instantaneous drift $\mu - \delta_t$ changes to $r - \delta_t$, yielding

$$\begin{aligned} P_t &= P_0 \exp\left(\int_0^t (r - \delta_u)\mathrm{d}u - \frac{1}{2}\sigma^2 t + \sigma B_t^{\mathbb{Q}}\right) && \text{(A5)} \\ &= P_0 \exp\left(\left(r - \mu + \alpha - \frac{1}{2}\sigma^2\right) t + \frac{\kappa}{\eta}\left(1 - \cos(\eta t)\right) + \sigma B_t^{\mathbb{Q}}\right), && \text{(A6)} \end{aligned}$$

where $B_t^{\mathbb{Q}}$ is a Brownian motion under the martingale measure.

Using Equation (A6), we can easily show that the martingale-measure expectation of the output price at time t taken at time s can be written as

$$\mathbb{E}_s^{\mathbb{Q}}[P_t] = P_s \exp\left((r-\mu+\alpha)(t-s) + \frac{\kappa}{\eta}\left(\cos(\eta s) - \cos(\eta t)\right)\right), \tag{A7}$$

allowing us, in turn, to write the first-order condition for maximization problem (6) as

$$P_t \exp\left(-\int_t^{t^*} (\mu - \alpha - \kappa \sin(\eta u))\mathrm{d}u\right)(\mu - \alpha - \kappa \sin(\eta t^*)) + c_I e^{-rt^*} = 0, \tag{A8}$$

which has to be numerically solved for t^*. Using the same equation, we can also calculate the amount of production output yielding a local maximum for objective function (7) from

$$Q_t' = \frac{\mathbb{E}_t^{\mathbb{Q}}[P_{t^*}]e^{-r(t^*-t)} - c_1 - C_I(t,t^*)}{c_2}. \tag{A9}$$

We ensure feasibility by setting $Q_t^* = \min\{\max\{0, Q_t'\}, \bar{K}\}$.

A.2 Finite Difference Scheme

We next discretize PDEs (8) and (9) on three and two-dimensional grids, respectively, approximate the partial derivatives in those PDEs using finite differences, and derive an explicit scheme relating firm value on some grid point to its values on other points. To begin with, we first replace the output price in both PDEs with its log counterpart $p_t = \ln(P_t)$. Doing so, PDE (8) becomes equal to

$$\frac{\partial W^{\mathrm{IB}}}{\partial t} + Q_t^* \frac{\partial W^{\mathrm{IB}}}{\partial I_t} + \left(r - \delta_t - \frac{1}{2}\sigma^2\right)\frac{\partial W^{\mathrm{IB}}}{\partial p_t} + \frac{1}{2}\sigma^2 \frac{\partial^2 W^{\mathrm{IB}}}{\partial p_t^2} - rW^{\mathrm{IB}} - c_1 Q_t^* - \frac{1}{2}c_2 Q_t^{*2} - c_I I_t = 0, \tag{A10}$$

while PDE (9) becomes equal to

$$\frac{\partial W^{\mathrm{IS}}}{\partial t} + \left(r - \delta_t - \frac{1}{2}\sigma^2\right)\frac{\partial W^{\mathrm{IS}}}{\partial p_t} + \frac{1}{2}\sigma^2 \frac{\partial^2 W^{\mathrm{IS}}}{\partial p_t^2} - rW^{\mathrm{IS}} + P_t Q_t^* - c_1 Q_t^* - \frac{1}{2}c_2 Q_t^{*2} = 0. \tag{A11}$$

We next consider the three-dimensional mesh on $[t_{\min}, t_{\max}] \times [p_{\min}, p_{\max}] \times [I_{\min}, I_{\max}]$ given by

$$\begin{cases} t_i = t_{\min} + i\Delta t; & i \ = 0, ..., N_t, \\ p_j = p_{\min} + j\Delta p; & j \ = 0, ..., N_p, \\ I_n = I_{\min} + n\Delta I; & n = 0, ..., N_I \end{cases} \tag{A12}$$

to discretize PDE (A10) and the two-dimensional mesh on $[t_{\min}, t_{\max}] \times [p_{\min}, p_{\max}]$ given by

$$\begin{cases} t_i = t_{\min} + i\Delta t; & i = 0, ..., N_t, \\ p_j = p_{\min} + j\Delta p; & j = 0, ..., N_p \end{cases} \tag{A13}$$

to discretize PDE (A11). In either case, we choose constant step sizes, $t_{\min} = 0$, $I_{\min} = 0$, and $p_{\min}$ sufficiently negative such that $P_{\min}$, the minimum non-logged output price, is close to zero. We further choose N_p such that the output price is within a five standard deviation window around its unconditional real-world expectation at time $t_{\max}$. Because inventory building periods never last longer than one seasonal cycle, we finally choose N_I such that the inventory axis goes up to $\bar{K}\frac{2\pi}{\eta}$, which is the output in inventory at the end of a cycle if the firm produced at full capacity over the entire cycle without selling output.

We now let $W^{\text{IB}}_{i,j,n}$ denote the value of $W^{\text{IB}}(t, P_t, I_t)$ at point (i, j, n) in grid (A12). We employ central and forward differences to approximate the partial derivatives in PDE (A10), assuming that the partial derivatives with respect to the log-price p_t and the inventory I_t have the same values on grid points (i, j, n) and $(i + 1, j, n)$ to obtain an explicit scheme. To be specific, we use the following approximations

$$\frac{\partial W}{\partial t} = \frac{W_{i+1,j,n} - W_{i,j,n}}{\Delta t} + \mathcal{O}(\Delta t), \tag{A14}$$

$$\frac{\partial W}{\partial I_t} = \frac{W_{i+1,j,n+1} - W_{i+1,j,n}}{\Delta I} + \mathcal{O}(\Delta I), \tag{A15}$$

$$\frac{\partial W}{\partial p_t} = \frac{W_{i+1,j+1,n} - W_{i+1,j-1,n}}{2\Delta p} + \mathcal{O}(\Delta p^2), \tag{A16}$$

$$\frac{\partial^2 W}{\partial p_t^2} = \frac{W_{i+1,j+1,n} - 2W_{i+1,j,n} + W_{i+1,j-1,n}}{\Delta p^2} + \mathcal{O}(\Delta p^2) \tag{A17}$$

in PDE (A10), leading the inventory builder's value at grid point (i, j, n) to be equal to

$$
\begin{aligned}
W^{\mathrm{IB}}_{i,j,n} = {} & \frac{1}{1+r\Delta t}\frac{\Delta t}{\Delta I}Q^*_{t_i}W^{\mathrm{IB}}_{i+1,j,n+1} - \frac{1}{1+r\Delta t}\frac{\Delta t}{\Delta I}Q^*_{t_i}W^{\mathrm{IB}}_{i+1,j,n} \\
& + \frac{1}{1+r\Delta t}\left(-\left(r-\delta_{t_i}-\frac{1}{2}\sigma^2\right)\frac{\Delta t}{2\Delta p} + \frac{1}{2}\sigma^2\frac{\Delta t}{\Delta p^2}\right)W^{\mathrm{IB}}_{i+1,j-1,n} \\
& + \frac{1}{1+r\Delta t}\left(1-\sigma^2\frac{\Delta t}{\Delta p^2}\right)W^{\mathrm{IB}}_{i+1,j,n} \\
& + \frac{1}{1+r\Delta t}\left(\left(r-\delta_{t_i}-\frac{1}{2}\sigma^2\right)\frac{\Delta t}{2\Delta p} + \frac{1}{2}\sigma^2\frac{\Delta t}{\Delta p^2}\right)W^{\mathrm{IB}}_{i+1,j+1,n} \\
& - \frac{1}{1+r\Delta t}\left(c_1 Q^*_{t_i} + \frac{1}{2}c_2 Q^{*2}_{t_i} + c_I I_n\right)\Delta t.
\end{aligned}
\tag{A18}
$$

Denoting by $W^{\mathrm{IS}}_{i,j}$ the value of $W^{\mathrm{IS}}(t, P_t)$ at point (i, j) in grid (A13), we also use Equations (A14), (A16), and (A17) in PDE (A11), leading the instantaneous seller's value at grid point (i, j) to be equal to

$$
\begin{aligned}
W^{\mathrm{IS}}_{i,j} = {} & \frac{1}{1+r\Delta t}\left(-\left(r-\delta_{t_i}-\frac{1}{2}\sigma^2\right)\frac{\Delta t}{2\Delta p} + \frac{1}{2}\sigma^2\frac{\Delta t}{\Delta p^2}\right)W^{\mathrm{IS}}_{i+1,j-1} \\
& + \frac{1}{1+r\Delta t}\left(1-\sigma^2\frac{\Delta t}{\Delta p^2}\right)W^{\mathrm{IS}}_{i+1,j} \\
& + \frac{1}{1+r\Delta t}\left(\left(r-\delta_{t_i}-\frac{1}{2}\sigma^2\right)\frac{\Delta t}{2\Delta p} + \frac{1}{2}\sigma^2\frac{\Delta t}{\Delta p^2}\right)W^{\mathrm{IS}}_{i+1,j+1} \\
& + \frac{1}{1+r\Delta t}\left(P_j Q^*_{t_i} - c_1 Q^*_{t_i} - \frac{1}{2}c_2 Q^{*2}_{t_i}\right)\Delta t.
\end{aligned}
\tag{A19}
$$

A.3 Recursive Solution and Boundary Conditions

We finally explain how we knit together the grids in Equations (A12) and (A13) to obtain the value of a firm which optimally decides to act as inventory builder in some time periods and as instantaneous seller in others. To that end, we solve for that firm's value recursively, starting at the terminal time $t_{\max}$ and assuming that the firm ceases to exist after that time. Choosing a sufficiently large $t_{\max}$, we can ensure that our finite difference estimate is arbitrarily close to the true value of the firm with an infinite horizon. While not strictly necessary, we also ensure that the firm is always an instantaneous seller at time $t_{\max}$.

We then solve the two-dimensional instantaneous-seller PDE (A11) on the entire two-dimensional grid in Equation (A13). To that end, we use the following boundary condition at $t = t_{\max}$

$$
W^{\mathrm{IS}}(t_{\max}, P_t) = \left(P_j Q^*_{t_{\max}} - C_P(Q^*_{t_{\max}})\right)\Delta t \approx 0,
\tag{A20}
$$

where $Q^*_{t_{\max}} = \min\left\{\max\left\{\frac{P_{t_{\max}} - c_1}{c_2}, 0\right\}, \bar{K}\right\}$. We can interpret that condition as the terminal instantaneous profit of an instantaneous seller before ceasing to exist. Realizing that $P_t = 0$ is an absorbing barrier for the stochastic process in Equation (1), we next use $W^{\text{IS}}(t, P_{\min}) = 0$ as boundary condition at $P = P_{\min}$. We finally realize that, as $P_t \to \infty$, the firm optimally produces at full capacity over its remaining lifetime, consistently setting $Q^*_t = \bar{K}$. We thus use the following boundary condition at $P = P_{\max}$

$$W^{\text{IS}}(t_i, P_{\max}) = \int_{t_i}^{t_{\max}} \left(\mathbb{E}^{\mathbb{Q}}_{t_i}[P_u]\bar{K} - C_P(\bar{K})\right) e^{-r(u-t_i)}\mathrm{d}u \tag{A21}$$

$$= P_{\max}\bar{K} \int_{t_i}^{t_{\max}} e^{-\int_{t_i}^{u} \delta_\tau \mathrm{d}\tau}\mathrm{d}u - \left(c_1\bar{K} + \frac{1}{2}c_2\bar{K}^2\right) \frac{1 - e^{-r(t_{\max}-t_i)}}{r}, \tag{A22}$$

noticing that the exterior integral requires a numerical solution. Relying on those boundary conditions, we can use Equation (A19) to fill in the entire two-dimensional grid in Equation (A13).

As a next step, we turn to the three-dimensional inventory-builder PDE (A10), solving it on the three-dimensional grid in Equation (A12) down until that output price below which the firm always acts as instantaneous seller (the lower output price boundary) and right until those output price-time combinations at which the firm conducts its final switch from inventory builder to instantaneous seller (the upper time boundary). In Figure 3, the lower output price boundary is, for example, close to 0.20, and the upper time boundary is the upward sloping part of the final parabola before the firm ceases to exist. We set the value of the firm on the lower output price boundary, $W^{\text{IB}}(t, P^l_t, I_t)$, equal to

$$W^{\text{IB}}(t, P^l_t, I_t) = W^{\text{IS}}(t, P^l_t), \tag{A23}$$

where P^l_t is the lower output price boundary, and $W^{\text{IS}}(t, P^l_t)$ is the value of the instantaneous seller at time t and output price P^l_t taken from the two-dimensional grid solved before. Conversely, we set the value of the firm on the upper time boundary, $W^{\text{IB}}(t^{\text{BtS}_n}, P^{\text{BtS}_n}_t, I_t)$, equal to

$$W^{\text{IB}}(t^{\text{BtS}_n}, P^{\text{BtS}_n}_t, I_t) = W^{\text{IS}}(t^{\text{BtS}_n}, P^{\text{BtS}_n}_t) + P_t I_t, \tag{A24}$$

where the t^{BtS_n}-$P^{\text{BtS}_n}_t$ pair is a time-output price combination at which the firm conducts its final switch from inventory builder to instantaneous seller, and $W^{\text{IS}}(t^{\text{BtS}_n}, P^{\text{BtS}_n}_t)$ is the value of the instantaneous seller at that combination taken from the two-dimensional grid solved before. Boundary condition (A24) implies that, on the optimal sales date, the value of the inventory builder is equal to the value of the corresponding instantaneous seller plus the sales revenue generated from liquidating the inventory.

While we again set the firm's value on the lower output-price boundary, $W^{\mathrm{IB}}(t, P_{\min}, I_t)$, equal to zero, we need to generalize our calculations in Equations (A21) and (A22) to find its value on the upper output-price boundary, $W^{\mathrm{IB}}(t, P_{\max}, I_t)$. To that end, we first recall that, as $P_t \to \infty$, the firm optimally produces at full capacity over its remaining lifetime, consistently setting $Q_t^* = \bar{K}$. For each grid point t_i on the upper output price boundary, we then use objective function (6) to identify all remaining inventory building and instantaneous sales regions until time $t_{\max}$ (see Figure 2 for an illustrative example). We finally set $W^{\mathrm{IB}}(t_i, P_{\max}, I_t)$ equal to the sum of the present values of the firm's net cash flows over each of those inventory building and instantaneous sales periods, including the remainder of the current period.

To calculate the time-s present value of the net cash flows generated by the firm over an inventory building period lasting from time t to t', with $s < t < t'$, we start with assuming that the firm holds no inventory at time t. In that case, the firm grows its inventory from zero to $\int_t^{t'} Q_u^* \mathrm{d}u = \bar{K}(t' - t)$ from time t to t', before depleting its entire inventory and selling $\bar{K}(t' - t)$ output units out of it at a price of $P_{t'}$ at time t'. Given that, the time-s present value of the net cash flows, $V_s^{\mathrm{IB}}(t, t')$, is

$$V_s^{\mathrm{IB}}(t,t') = \mathbb{E}_s^{\mathbb{Q}}[P_{t'}]\bar{K}(t'-t)e^{-r(t'-s)} - \int_t^{t'} C_P(\bar{K})e^{-r(u-s)}\mathrm{d}u - \int_t^{t'} \bar{K}(u-t)c_I e^{-r(u-s)}\mathrm{d}u \tag{A25}$$

$$\begin{aligned} &= P_{\max}\bar{K}(t'-t)e^{-\int_s^{t'}\delta_u \mathrm{d}u} - \left(c_1\bar{K} + \frac{1}{2}c_2\bar{K}^2\right)\frac{e^{-r(t-s)} - e^{-r(t'-s)}}{r} \\ &\quad - \frac{c_I}{r^2}\bar{K}\left(e^{-r(t-s)} - e^{-r(t'-s)}(1 + r(t'-t))\right), \end{aligned} \tag{A26}$$

where $\int_s^{t'} \delta_u \mathrm{d}u = (\mu - \alpha)(t' - s) + \frac{\kappa}{\eta}\left(\cos(\eta t') - \cos(\eta s)\right)$. If, in contrast, the firm already holds an amount of inventory equal to $\bar{I}_t$ at time t, we need to add the present value of the incremental cash flows associated with that inventory to $V_s^{\mathrm{IB}}(t, t')$. We can calculate that present value, $V_s^{\mathrm{IB}+}(t, t')$, using

$$V_s^{\mathrm{IB}+}(t,t') = \mathbb{E}_s^{\mathbb{Q}}[P_{t'}]\bar{I}_t e^{-r(t'-s)} - \int_t^{t'} \bar{I}_t c_I e^{-r(u-s)}\mathrm{d}u \tag{A27}$$

$$= \bar{I}_t\left(P_{\max}e^{-\int_s^{t'}\delta_u \mathrm{d}u} - c_I\frac{e^{-r(t-s)} - e^{-r(t'-s)}}{r}\right). \tag{A28}$$

To calculate the time-s present value of the net cash flows generated by the firm over an instantaneous selling period lasting from time t to t', with again $s < t < t'$, we first recall that, over each instant within that period, the firm produces an amount of output equal to $Q^*_u \mathrm{d}u = \bar{K}\mathrm{d}u$ and instantaneously sells that at a price of P_u. Given that, the time-s present value of the net cash flows, $V^{\mathrm{IS}}_s(t, t')$, is

$$V^{\mathrm{IS}}_s(t,t') = \int_t^{t'} \left(\mathbb{E}^{\mathbb{Q}}_s[P_u]\bar{K} - C_P(\bar{K})\right) e^{-r(u-s)}\mathrm{d}u \tag{A29}$$

$$= P_{\max}\bar{K}\int_t^{t'} e^{-\int_s^u \delta_\tau \mathrm{d}\tau}\mathrm{d}u - \left(c_1\bar{K} + \frac{1}{2}c_2\bar{K}^2\right)\frac{e^{-r(t-s)} - e^{-r(t'-s)}}{r}. \tag{A30}$$

We finally find the firm's value on the upper, $W^{\mathrm{IB}}(t, P_t, I_{\max})$, and on the lower inventory boundary, $W^{\mathrm{IB}}(t, P_t, I_{\min})$. To do so, we start from the forward finite difference approximation

$$\frac{\partial W^{\mathrm{IB}}(t_i, P_t, I_t)}{\partial t} \approx \frac{W^{\mathrm{IB}}(t_{i+1}, P_t, I_t) - W^{\mathrm{IB}}(t_i, P_t, I_t)}{\Delta t}. \tag{A31}$$

We then notice that, keeping the output price, P_t, and the amount of inventory, I_t, constant, the difference in firm value between time t_i and t_{i+1} is the present value of the additional output produced and put into inventory over that period. That present value is approximately equal to

$$W^{\mathrm{IB}}(t_{i+1}, P_t, I_t) - W^{\mathrm{IB}}(t_i, P_t, I_t) = -\left(\mathbb{E}^{\mathbb{Q}}_{t_i}[P_{t^*}]Q^*_{t_i}e^{-r(t^*-t_i)} - C_P(Q^*_{t_i}) - Q^*_{t_i}C_I(t_i, t^*)\right)\Delta t, \tag{A32}$$

allowing us to write the firm's value on the upper inventory boundary, $W^{\mathrm{IB}}(t, P_t, I_{\max})$, as

$$W^{\mathrm{IB}}(t_i, P_t, I_{\max}) = W^{\mathrm{IB}}(t_{i+1}, P_t, I_{\max}) + \left(\mathbb{E}^{\mathbb{Q}}_{t_i}[P_{t^*}]Q^*_{t_i}e^{-r(t^*-t_i)} - C_P(Q^*_{t_i}) - Q^*_{t_i}C_I(t_i, t^*)\right)\Delta t, \tag{A33}$$

and the firm's value on the lower inventory boundary, $W^{\mathrm{IB}}(t, P_t, I_{\min})$, as

$$W^{\mathrm{IB}}(t_i, P_t, I_{\min}) = W^{\mathrm{IB}}(t_{i+1}, P_t, I_{\min}) + \left(\mathbb{E}^{\mathbb{Q}}_{t_i}[P_{t^*}]Q^*_{t_i}e^{-r(t^*-t_i)} - C_P(Q^*_{t_i}) - Q^*_{t_i}C_I(t_i, t^*)\right)\Delta t. \tag{A34}$$

Having used Equation (A18) in conjunction with the above boundary conditions to fill in the three-dimensional grid, we finally always replace $W^{\mathrm{IB}}(t, P_t, I_t)$ with $P_tI_t + W^{\mathrm{IB}}(t, P_t, 0)$ when $P_tI_t + W^{\mathrm{IB}}(t, P_t, 0) > W^{\mathrm{IB}}(t, P_t, I_t)$. Doing so, we ensure that the firm always immediately sells off its entire output in inventory when it is value-maximizing to do so. We further ensure that firm value cannot turn negative.

We then return to the two-dimensional instantaneous-seller PDE (A11), solving it on the two-dimensional grid in Equation (A13) down until that output price below which the firm always acts as instantaneous seller (the lower output price boundary) and right until those output price-time combinations at which the firm conducts its final switch from instantaneous seller to inventory builder (the upper time boundary). While the lower output price boundary in Figure 3 is, for example, again close to 0.20, the upper time boundary is now the downward sloping part of the final parabola before the firm ceases to exist. While we else use the same boundary conditions as for the prior two dimensional grid, we now set the firm's value on the upper time boundary at which the firm switches from instantaneous seller to inventory builder to

$$W^{\mathrm{IS}}(t^{\mathrm{StB}_n}, P^{\mathrm{StB}_n}) = W^{\mathrm{IB}}(t^{\mathrm{StB}_n}, P_t^{\mathrm{StB}_n}, I_t = 0), \tag{A35}$$

where the t^{StB_n}-$P_t^{\mathrm{StB}_n}$ pair is a time-output price combination at which the firm switches, and $W^{\mathrm{IB}}(t^{\mathrm{StB}_n}, P_t^{\mathrm{StB}_n}, I_t = 0)$ is the value of an inventory builder holding zero inventory. As before, we take the lower output price and upper time boundary values from the solution to the prior three dimensional grid.

We continue in that manner, next solving the three-dimensional inventory-builder PDE (A10) on the three-dimensional grid in Equation (A12) down until that output price below which the firm always acts as instantaneous seller (the lower output price boundary) and right until those output price-time combinations at which the firm conducts its penultimate switch from inventory builder to instantaneous seller (the upper time boundary). We again take the lower output price and upper time boundary values from the solution to the prior two dimensional grid. Having solved all available grids, we finally merge them into one firm value grid, always collecting firm value during instantaneous selling periods from the appropriate two-dimensional grids and during inventory building periods from the appropriate three-dimensional grids.

We compute the firm's expected excess return as in Equation (12). To this end, let $ER_{i,j,n}$ denote that return at grid point (i, j, n) in the firm value grid. Approximating the partial derivative in Equation (12) using central differences, we can calculate the expected excess return at grid point (i, j, n) as

$$ER_{i,j,n} = \frac{W_{i,j+1,n} - W_{i,j-1,n}}{P_{j+1} - P_{j-1}} \frac{P_j}{W_{i,j,n}}(\mu - r). \tag{A36}$$

B Growth Option Extension

In this appendix, we endow the firm in our main theoretical model with a single growth option allowing it to instantaneously and irreversibly double its production capacity at an investment cost of k. In technical jargon, the growth option is thus a perpetual American call option written on $\bar{K}$ additional production units. To make sure that the firm's total production costs continue to be described by the convex function $c_1 Q_t + \frac{1}{2} Q_t^2$, we assume that the cost of producing Q_t output units with the additional production units is $C_P(Q_t) = c_1(\bar{K} + Q_t) + \frac{1}{2} c_2 (\bar{K} + Q_t)^2$, with $Q_t \in [0, \bar{K}]$. Under that assumption, we are able to compute the value of the additional production units, which we denote by $U(t, P_t, I_t)$, using the methodology in Appendix A, using slightly amended versions of maximization problem (7), the upper output price boundary condition derived in Equations (A26), (A28), and (A30) in Appendix A, and the upper and lower inventory boundary conditions in, respectively, Equations (A33) and (A34) in that appendix.

We next value the growth option written on the $\bar{K}$ additional production units. Given that exercising the option yields production units with an empty inventory, the value of the growth option is independent of the output in inventory I_t, allowing us to write it as $F = F(t, P_t)$. As generally the case for American call options, the firm optimally exercises the growth option whenever the output price P_t rises above the time-varying threshold P_t^*. Before the firm exercises the growth option, it is then easy to show that the value of the growth option has to satisfy the two-dimensional PDE

$$\frac{\partial F}{\partial t} + \left(r - \delta_t - \frac{1}{2}\sigma^2 \right) \frac{\partial F}{\partial p_t} + \frac{1}{2}\sigma^2 \frac{\partial^2 F}{\partial p_t^2} - rF = 0. \tag{B1}$$

We solve PDE (B1) subject to the usual spatial boundary conditions for American call options, $F(t_i, P_{\min}) = 0$ and $F(t_i, P_{\max}) = U(t_i, P_{\max}, 0) - ke^{-r(t_{\max} - t_i)}$ as well as the terminal boundary condition $F(t_{\max}, P_j) = \max\{U(t_{\max}, P_j, 0) - k, 0\}$. We further determine the free exercise boundary, P_t^*, from

$$F(t, P_t^*) = U(t, P_t^*, 0) - k. \tag{B2}$$

To solve PDE (B1) using an explicit finite difference scheme, we let $F_{i,j}$ be the value of $F(t, P_t)$ at point (i, j) in grid (A13). We next plug Equations (A14), (A16), and (A17) into PDE (B1), allowing us to

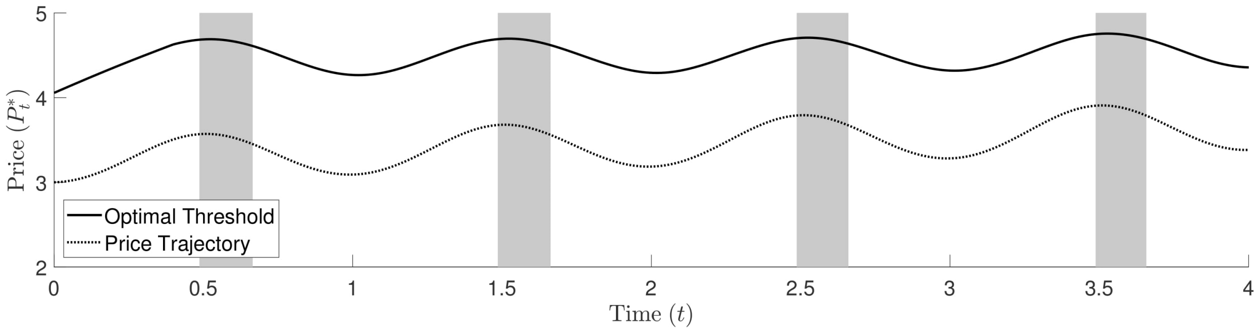

Figure B.1: The figure plots the optimal output price threshold P_t^* above which the firm exercises its growth option over the period from $t = 0$ to 4 under an output price trajectory at which firm value shows no general tendency to rise or fall. The vertical line shows the exercise time. The gray bars indicate the periods during which the firm acts as an instantaneous seller. We describe the parameter values in the text.

calculate the option's value at grid point (i, j) from

$$\begin{aligned} F_{i,j} &= \frac{1}{1+r\Delta t}\left(-\left(r-\delta_{t_i}-\frac{1}{2}\sigma^2\right)\frac{\Delta t}{2\Delta p}+\frac{1}{2}\sigma^2\frac{\Delta t}{\Delta p^2}\right)F_{i+1,j-1} \\ &+ \frac{1}{1+r\Delta t}\left(1-\sigma^2\frac{\Delta t}{\Delta p^2}\right)F_{i+1,j} \\ &+ \frac{1}{1+r\Delta t}\left(\left(r-\delta_{t_i}-\frac{1}{2}\sigma^2\right)\frac{\Delta t}{2\Delta p}+\frac{1}{2}\sigma^2\frac{\Delta t}{\Delta p^2}\right)F_{i+1,j+1}. \end{aligned} \tag{B3}$$

To incorporate option exercises, we replace each $F_{i,j}$ value in the grid with the corresponding exercise payoff $\max\{U(t_i, P_j, 0) - k, 0\}$ if the exercise payoff exceeds the original value. We finally set P_t^* to the lowest output price in the grid for which the exercise payoff exceeds the original value per time t.

The value of the firm, $W(t, P_t, I_t)$, then becomes the sum of its production capacity-in-place, $V(t, P_t, I_t)$, valued as described in Appendix A, and its growth option, $F(t, P_t)$

$$W(t, P_t, I_t) = V(t, P_t, I_t) + F(t, P_t), \tag{B4}$$

whereas its conditional expected excess return, $\mathbb{E}[r_W] - r$, turns into the value-weighted average of the expected return on the production capacity-in-place and the growth option

$$\mathbb{E}[r_W] - r = \left(\frac{V(t, P_t, I_t)}{W(t, P_t, I_t)}\Omega_V + \frac{F(t, P_t)}{W(t, P_t, I_t)}\Omega_F\right)(\mu - r), \tag{B5}$$

where $\Omega_F = \frac{\partial F(t,P_t)}{\partial P_t}\frac{P_t}{F(t,P_t)}$ is the elasticity of the growth option.

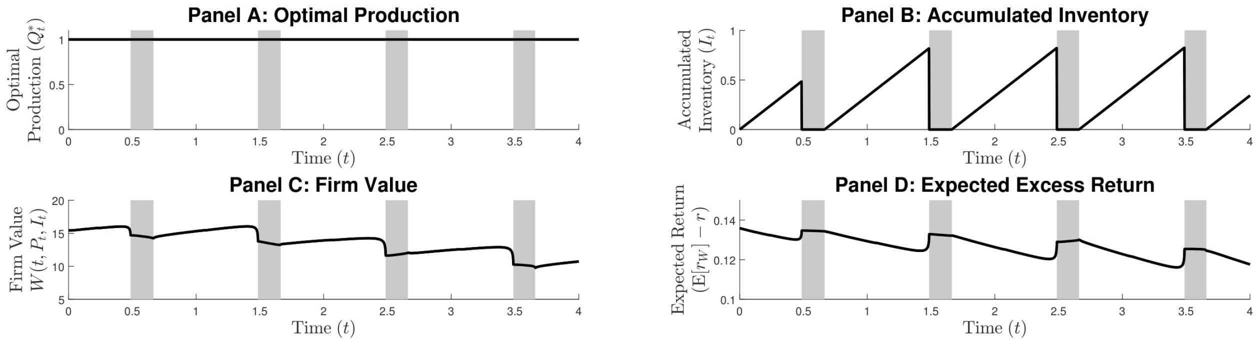

Figure B.2: The figure plots the firm's optimal production quantity Q_t^* (Panel A), its accumulated output in inventory I_t (Panel B), its value $W(t, P_t, I_t)$ (Panel C), and its expected excess return $\mathbb{E}[r_W] - r$ (Panel D) over the period from $t = 0$ to 4 under an output price trajectory at which firm value shows no general tendency to rise or fall. The gray bars in each subplot indicate the periods during which the firm acts as an instantaneous seller. We describe the parameter values in the text.

Figure B.1 plots the optimal output price threshold above which the firm exercises its growth option P_t^* (solid black line) under the output price trajectory also used in the main paper (broken black line) over the time period from $t = 0$ to 4. We use the same parameter values as in Section 2.2 to construct the figure. The exception is the starting value of the output price, P_0, which we set to three to ensure that the growth option captures a meaningful fraction of total firm value. We set the investment cost, k, to two. The figure suggests that the seasonality in the output price translates into seasonality in the optimal output price threshold. Interestingly, however, it further shows that the firm is more likely to invest during a low rather than high output price season (i.e., the output price threshold is high when the seasonal output price is high, and vice versa). The reason is that adding new production capacity during a low output price season allows the firm to raise its production over that season to sell more output over the next high output price season, boosting its profitability and, as a consequence, maximizing its value.

Figure B.2 plots the optimal production quantity Q_t^* (Panel A), accumulated inventory I_t (Panel B), value (Panel C), and expected excess return (Panel D) of the firm with a growth option at the same output price trajectory as above over the time period from $t = 0$ to 4. Given that we assume a higher starting value for the output price, P_0, to ensure that the growth option captures a meaningful fraction of total firm value, Panel A shows that the firm consistently produces at its full capacity $\bar{K}$. In accordance, Panel B shows that the firm's accumulated inventory rises more rapidly than in Panel B in Figure 5 in the main text, while Panel C indicates that firm value is higher than in Panel C in that same figure. Notwithstanding the growth option, Panel D, however, shows that the expected excess return of the firm with growth option behaves similarly over time as that of the firm without growth option (compare with Panel D in 5). To be precise, the expected excess return still markedly rises over inventory building periods, shoots up at their end, but then stays close to constant over instantaneous selling periods.

C Variable Definitions

Table C1
Variable Definitions
The table presents the definitions of our analysis variables. In our asset pricing tests, we update the variables indexed by "M" ("Q") ["A"] on a monthly (quarterly) [annual] basis and use their values to condition returns over month $t+1$ (month $t+1$) [the period from July of year t to June of year $t+1$]. We show the data-provider (CRSP and Compustat) mnemonics of the variables in parentheses.

Variable Name	Variable Definition
	Panel A: Seasonality Variables
QSales (Q)	Mean of sales proportion of the current quarter in year $t-2$ and $t-3$. The sales proportion equals quarterly sales (saleq) divided by sum of all quarterly sales in that fiscal year (see Grullon et al. (2020)).
QSalesRank (Q)	Monthly rank of *QSales* scaled by the monthly number of observations.
Seasonality (A)	Standard deviation of the four *QSales* values within one fiscal year.
QInventory (Q)	Mean of inventory proportion of the previous quarter in year $t-2$ and $t-3$. The proportion equals quarterly total inventory (invtq)) divided by sum of all quarterly inventory in that fiscal year.
DummyInventoryBuilder (A)	Dummy equal to one if *QSales* and *QInventory* take their maximum value in the same quarter of that year, else zero.
QFGInventory (Q)	Mean of inventory proportion of the previous quarter in year $t-2$ and $t-3$. The proportion equals quarterly finished goods inventory (invfgq) divided by sum of all quarterly inventory in that fiscal year.
	Panel B: Control Variables
MarketBeta (M)	Sum of slope coefficients from a stock-level regression of excess stock returns (ret) on current, one-day lagged, and the sum of two-, three-, and four-day lagged excess market returns, where the regression is run using daily data over the prior 12 months. We require that the regression is run on at least 200 observations (see Lewellen and Nagel (2006)).
MarketSize (A)	Log of the product of the stock price (abs(prc)) and common shares outstanding (shrout) at the end of the prior calender year (in millions).
BookToMarket (A)	Log of the ratio of the book value of equity to the market value of equity (abs(prc) times shrout), where the book value of equity equals stockholder's equity (seq) plus deferred taxes (txditc) plus investment tax credit (itcb, zero if missing) minus preferred stock (pstkrv, pstkl, pstk, or zero, in that order of availability). The variables are from the fiscal year-end in calendar year $t-1$ (see Fama and French (1992, 1993)).

(continued on next page)

Table C1
Variable Definitions (cont.)

Variable Name	Variable Definition
Momentum (M)	Log of one plus the stock return (ret) compounded over the period from month $t-11$ to month $t-1$ (see Jegadeesh and Titman (1993)).
Investment (A)	Log of the gross percentage change in total assets (at) from the fiscal year-end in calendar year $t-2$ to the fiscal year-end in year $t-1$ (see Fama and French (2015)).
Profitability (A)	Ratio of sales (sale) net of costs of goods sold (cogs), selling, general, and administrative expenses (xsga, zero if missing), and interest expenses (xint, zero if missing) to the book value of equity, which equals stockholder's equity (seq) plus deferred taxes (txditc) plus investment tax credit (itcb, zero if missing) minus preferred stock (pstkrv, pstkl, pstk, or zero, in that order of availability). The variables are from the fiscal year-end in calendar year $t-1$ (see Fama and French (2015)).
	Panel C: Portfolio Characteristics
RSeason(xy) (M)	Mean of the same calendar-month return (ret) taken over the prior x calendar years, with x equal to three, five, and seven (see Heston and Sadka (2008)).
ESeason (Q)	Average rank of the current fiscal quarter in a ranking of the last 20 quarterly earnings (measured as earnings per share excluding extraordinary items adjusted for stock splits (epsfxq)) from highest to lowest (see Chang et al. (2017)).